Soft-Collinear Effective Theory

Part II: Scalar SCET

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Outline

Part I: The strategy of regions

Part II: Scalar SCET

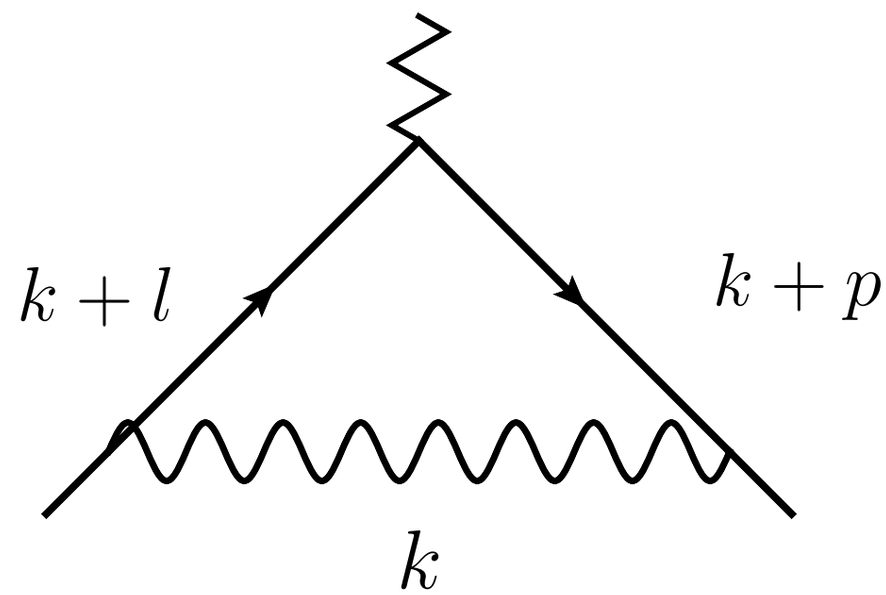
Part III: Generalization to QCD

Part IV: Resummation by RG evolution

Part V: IR divergences of gauge theory amplitudes

Recap

In the first lecture, we discussed the "strategy of regions" technique and applied it to the Sudakov problem, the form factor with $Q^2 \gg L^2 \sim P^2$.



$$L^2 \equiv -l^2 - i0\,, \qquad P^2 \equiv -p^2 - i0\,, \qquad Q^2 \equiv -(l-p)^2 - i0$$

We found that four momentum regions contribute to the expansion in $\lambda^2 \sim P^2/Q^2 \sim L^2/Q^2$

	$(n\cdot k\,,\,\bar{n}\cdot k\,,\,k_\perp)$
• Hard (h)	$k^\mu \sim \big(1\ \ ,\ 1\ \ ,\ 1\ \big)\,Q$
• Collinear to p (c1)	$k^\mu \sim \big(\lambda^2\ ,\ 1\ \ ,\ \lambda\ \big)\,Q$
• Collinear to l (c2)	$k^\mu \sim \big(1\ \ ,\ \lambda^2\ ,\ \lambda\ \big)\,Q$
• Soft (s)	$k^\mu \sim \big(\lambda^2\ ,\ \lambda^2\ ,\ \lambda^2\big)\,Q$

where $n_\mu = (1,0,0,1)$ and $\bar{n}_\mu = (1,0,0,-1)$ are light-cone reference vector in the directions of p_μ and l_μ.

The contributions from the different regions are obtained by assuming a given scaling for the loop momentum and then expanding the *integrand* accordingly. For the soft region, for example, we got

$$I_{\rm s} = i\pi^{-d/2}\mu^{4-d}\int d^dk\,\frac{1}{(k^2+i0)\,(2k_-\cdot l_+ + l^2 + i0)\,(2k_+\cdot p_- + p^2 + i0)}$$

We now construct an effective Lagrangian whose Feynman rules directly generate the expanded diagrams.

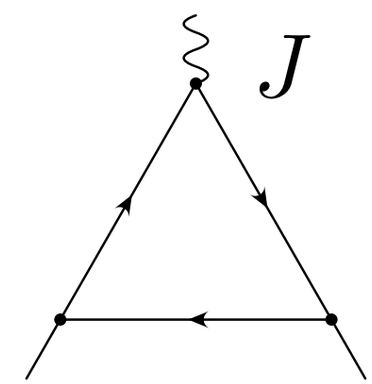
Scalar SCET

We now construct an effective Lagrangian whose Feynman rules yield exactly the different contributions found in the strategy of regions.

Since we were looking at scalar diagrams, let's work with the scalar theory (in *d* dimensions)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{g}{3!}\phi^3(x)$$

with external current $J(x) = \phi^2(x)$



The important elements in the construction of the EFT for QCD are the same, but because the different quark and gluon field components scale differently, the Lagrangian looks more complicated.

To obtain the SCET Lagrangian, we split the field into

$$\phi(x) \longrightarrow \phi_{c1}(x) + \phi_{c2}(x) + \phi_s(x)$$

Note that we did not introduce a field for the hard region: the hard contributions are absorbed into Wilson coefficients (i.e. coupling constants) multiplying operators built from soft and collinear fields.

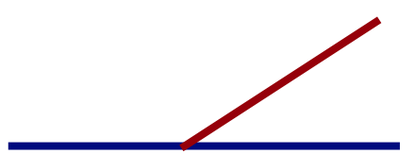
In order for the construction to make sense, we should introduce external sources for the three fields which scale in the appropriate way.

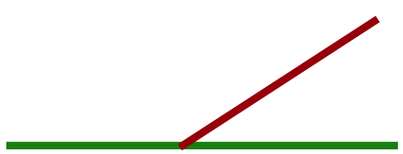
Inserting into the Lagrangian, we get

$$\mathcal{L} \to \mathcal{L}_{c1} + \mathcal{L}_{c2} + \mathcal{L}_{s} + \mathcal{L}_{\mathrm{c+s}}$$

three copies of the original Lagrangian as well as terms which contain soft and collinear interactions:

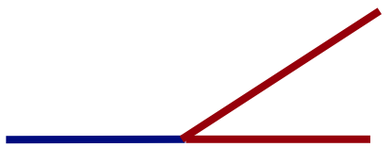
$$\mathcal{L}_{\mathrm{c+s}} = -\frac{g}{2}\phi_{c1}^2(x)\phi_s(x) - \frac{g}{2}\phi_{c2}^2(x)\phi_s(x)$$





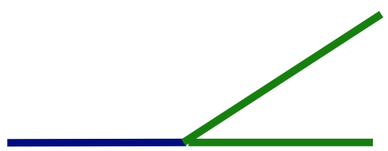
All other terms are forbidden by momentum conservation.

Terms forbidden by momentum conservation:



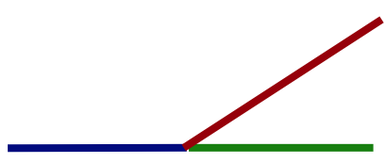
$$\phi_{c1}(x)\phi_s^2(x)$$

An energetic particle cannot decay into soft particles only.



$$\phi_{c1}(x)\phi_{c2}^2(x)$$

An particle flying in the +z direction cannot decay into two particles moving in the negative z direction.



$$\phi_{c1}(x)\phi_{c2}(x)\phi_s(x)$$

Derivative (or "multipole") expansion

Beneke and Feldmann hep-ph/0211358

As a final step, we need to expand in small momentum components.

$$\int d^d x\, \phi_{c1}^2(x)\phi_s(x) = \int d^d x \int \frac{d^d p_1}{(2\pi)^d} \int \frac{d^d p_2}{(2\pi)^d} \int \frac{d^d p_s}{(2\pi)^d}\, \phi_{c1}^2(p_1)\, \phi_{c1}^2(p_2)\, \phi_s(p_s)\, e^{-i(p_1+p_2+p_s)x}$$

$p_1^\mu + p_2^\mu + p_s^\mu$ scales as $(\lambda^2, 1, \lambda)$

x^μ scales as $(1, 1/\lambda^2, 1/\lambda)$

p_s^μ scales as $(\lambda^2, \lambda^2, \lambda^2)$

So $p_s \cdot x = 2p_{s+} \cdot x_- + 2p_{s-} \cdot x_+ + p_{s\perp} \cdot x_\perp$

$$O(1) + O(\lambda^2) + O(\lambda)$$

Derivative expansion

So we can expand in small derivative terms:

$$[\ 1 + \quad O(\lambda) \ + O(\lambda^2) + \ldots\ ]$$

$$\int d^d x\, \phi_{c1}^2(x)\phi_s(x) = \int d^d x\ \phi_{c1}^2(x)\left[1 + x_\perp \cdot \partial_\perp + x_+ \cdot \partial_{x_-} + \ldots\right]\ \phi_s(x)|_{x=x_-}$$

$$= \int d^d x\ \phi_{c1}^2(x)\phi_s(x_-) + \ldots$$

Note that the expanded Lagrangian is only translation invariant up to higher order terms.

An alternative formalism to perform the expansion treats the large momenta as labels on the fields.

Final result: leading-power scalar SCET Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu \phi_{c1}(x)\partial^\mu \phi_{c1}(x) - \frac{g}{3!}\phi_{c1}^3(x)$$

$$+ \frac{1}{2}\partial_\mu \phi_{c2}(x)\partial^\mu \phi_{c2}(x) - \frac{g}{3!}\phi_{c2}^3(x)$$

$$+ \frac{1}{2}\partial_\mu \phi_{s}(x)\partial^\mu \phi_{s}(x) - \frac{g}{3!}\phi_{s}^3(x)$$

$$- \frac{g}{2}\phi_{c1}^2(x)\phi_s(x_-) - \frac{g}{2}\phi_{c2}^2(x)\phi_s(x_+)$$

$$x_-^\mu = x \cdot \bar{n}\frac{n_\mu}{2}$$

Normally the hard contributions lead to matching corrections. To take them into account, one has to write down the most general form of the Lagrangian, i.e. all operators compatible with the symmetries of the theory, multiplied by arbitrary coefficients.

These Wilson coefficients are then determined by matching:

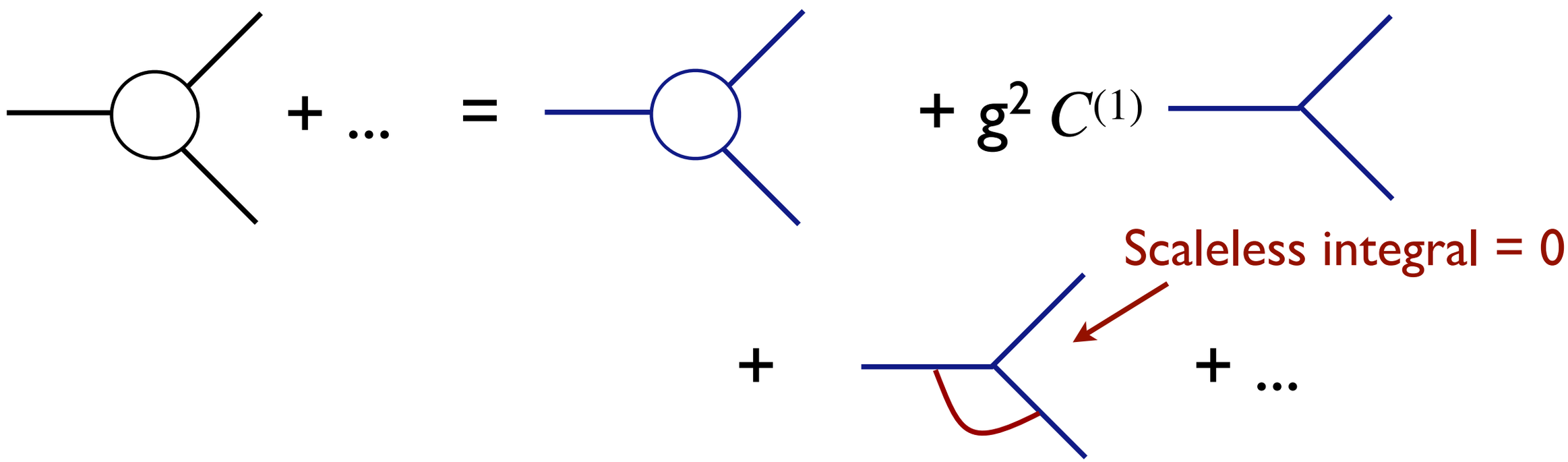
- Calculate the same quantity in the full and effective theory.
- Adjusts the Wilson coefficients so as to reproduce the full theory result.

No matching corrections for $\mathcal{L}_{\text{eff}}$

Introduce Wilson coefficients, e.g. arbitrary coefficient of the $\phi_{c1}{}^3$ term.

$$\mathcal{L}_{c1} = \frac{1}{2}\partial_\mu \phi_{c1}(x)\partial^\mu \phi_{c1}(x) - \frac{g}{3!}\, C\, \phi_{c1}^3(x)$$ with $$C = 1 + g^2 C^{(1)} + g^4 C^{(2)} + \ldots$$

Matching calculation for $C^{(1)}$:



$\rightarrow$ C=1 to all orders in PT.

Current operator

In contrast to $\mathcal{L}_{\text{eff}}$, the current operator J *does* receive matching corrections.

Write down the most general form in the effective theory, perform matching calculation.

$$J = J_2 + J_3 + \cdots \sim C_2\, \phi_{c1}\phi_{c2} + C_3\, [\phi_{c1}^2\phi_{c2} + \phi_{c1}\phi_{c2}^2] + \ldots$$

We also have to allow for derivative terms.

$$n\cdot\partial\,\phi_{c1}(x) \sim \lambda^2\phi_{c1}(x)\,, \quad \bar{n}\cdot\partial\,\phi_{c1}(x) \sim \lambda^0\phi_{c1}(x)\,, \quad \partial_\perp^\mu\,\phi_{c1}(x) \sim \lambda\,\phi_{c1}(x)$$

unsuppressed!

The derivatives $\bar{n} \cdot \partial \, \phi_{c1}(x)$and $n \cdot \partial \, \phi_{c2}(x)$ are unsuppressed, because the collinear fields carry large energies in these directions. Even at leading power, we need to allow for arbitrary many such derivatives!

Or, in other words, we have to allow for non-locality of the collinear fields along these directions:

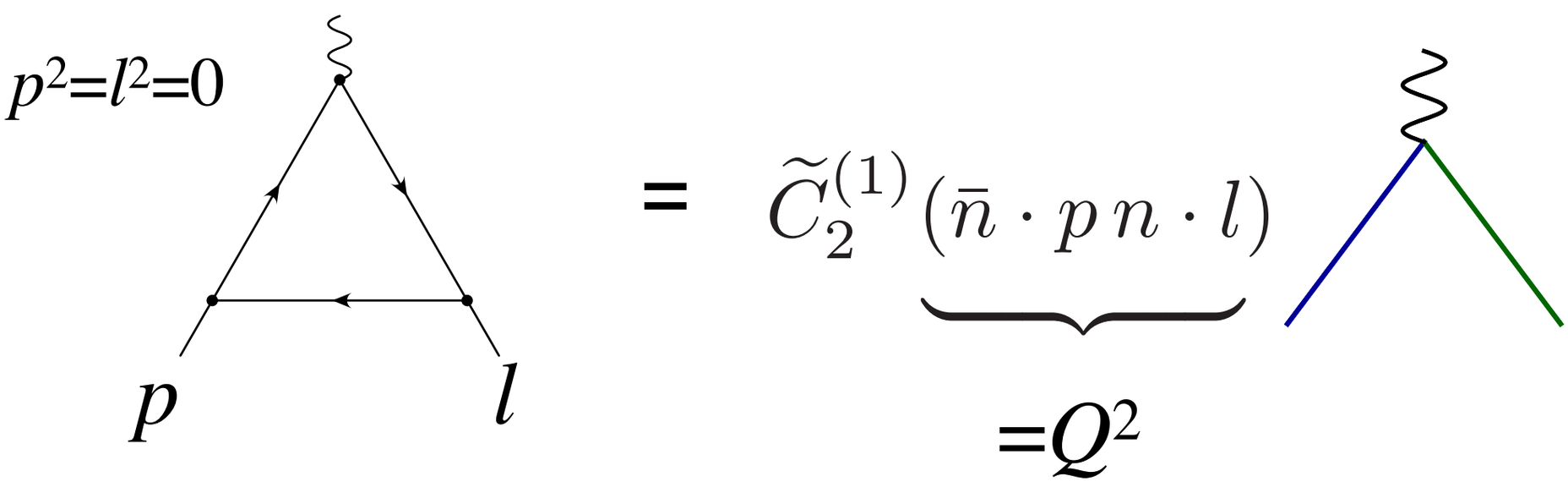
$$J_2(x) = \int ds \, dt \, C_2(s,t) \, \phi_{c1}(x + s\bar{n}) \phi_{c2}(x + tn)$$

SCET operators are non-local along light-cone directions corresponding to large energies.

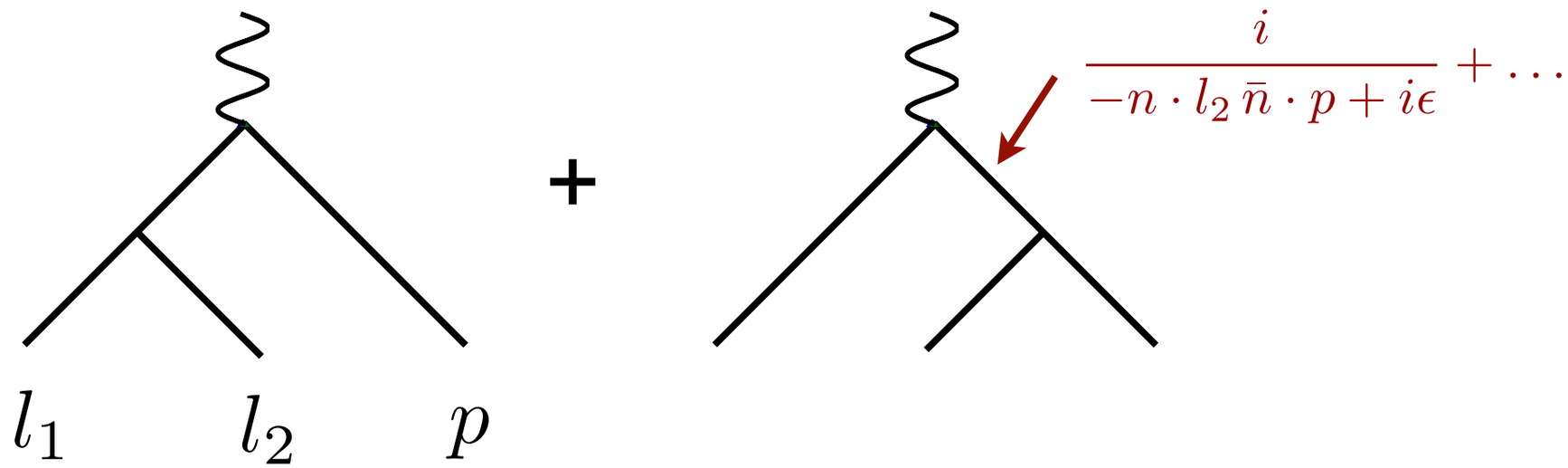
This non-locality in position space translates into dependence on the large energies in momentum space

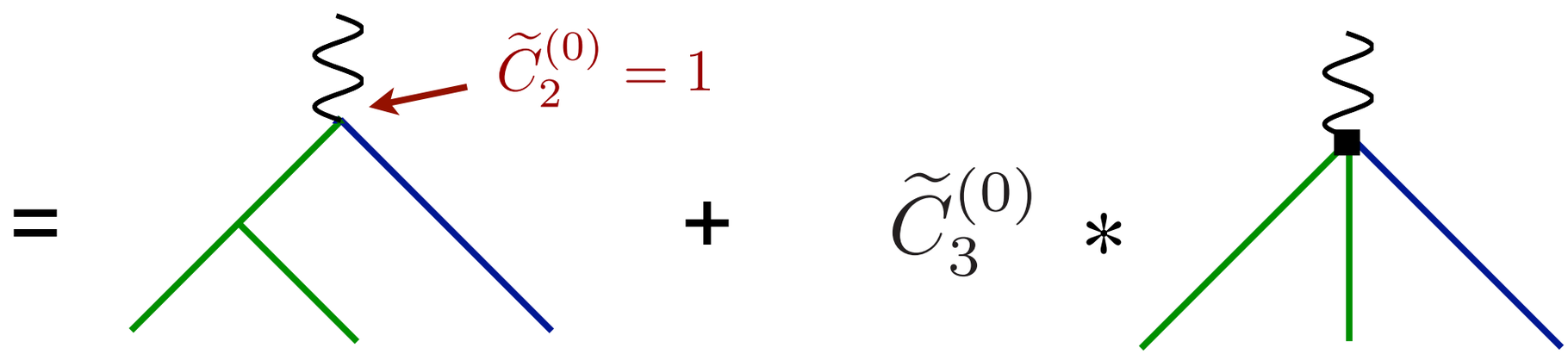
$$\widetilde{C}_2(\bar{n}\cdot p\, n\cdot l) = \int dsdt\, e^{is\bar{n}\cdot p}\, e^{-itn\cdot l} C_2(s,t)$$

One-loop matching



Tree-level matching for J_3

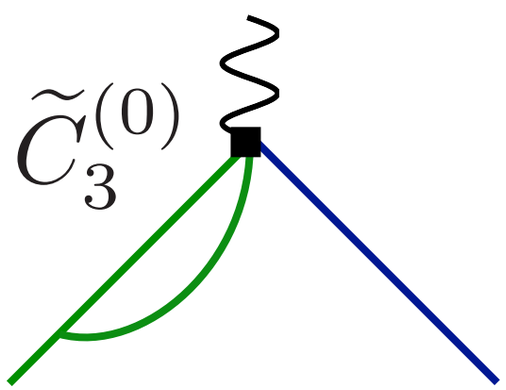




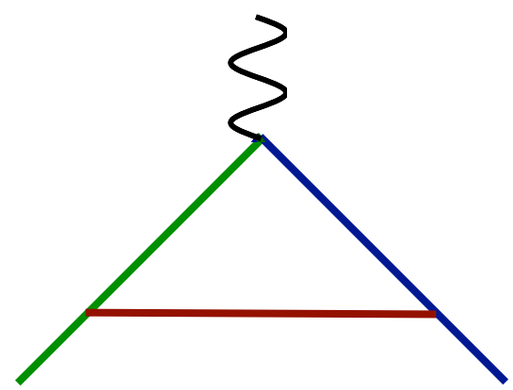
$$\widetilde{C}_3^{(0)}(n\cdot l_1, n\cdot l_2, \bar{n}\cdot p) = \frac{-g}{-n\cdot l_2\,\bar{n}\cdot p + i\epsilon}$$

With the effective Lagrangian and the tree-level current operator, we can now calculate the one-loop corrections to the three-point functions.

The one-loop diagrams of the effective theory are in one-to-one correspondance to the loop integrals encountered using the strategy of regions method.

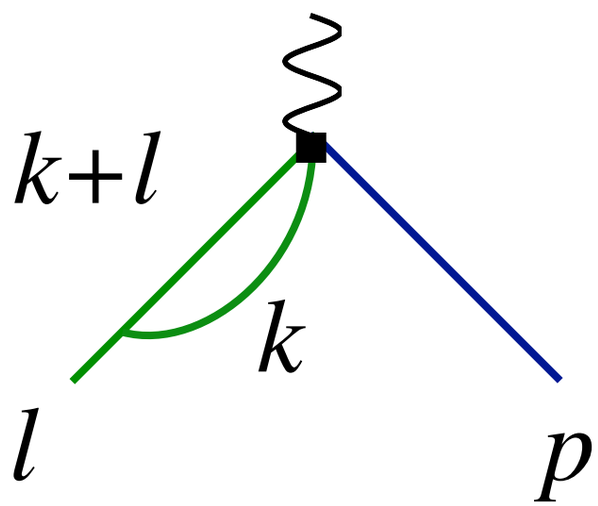


region c2



region s

Collinear region



$$\sim \int d^d k \, \frac{1}{(k^2 + i0)\,[(k+l)^2 + i0]\,(2k_+ \cdot p_- + i0)}$$

$$\widetilde{C}_3^{(0)}$$

Excercise: show that the diagram with the soft exchange indeed gives the contribution from the soft region.

Power counting in d=6

The kinetic terms in the action are O(λ^0)

$$\mathcal{S}_{\text{eff}} = \int d^6x \, \frac{1}{2}\partial_\mu \phi_s(x)\partial^\mu \phi_s(x) + \int d^6x \, \frac{1}{2}\partial_\mu \phi_{c1}(x)\partial^\mu \phi_{c1}(x) + \ldots$$

$$\lambda^{-12} \qquad\qquad \lambda^{-6}$$

which implies $\phi_s(x) \sim \lambda^4$

$\phi_c(x) \sim \lambda^2$

The fields have mass dimension 2.

Interactions

For the interaction terms, we have:

$$\mathcal{S}_{\rm int} = -\frac{g}{3!}\int d^6x\,\phi_{c2}^3(x) - \frac{g}{3!}\int d^6x\;\phi_s^3(x) - \frac{g}{2}\int d^6x\,\phi_{c1}^2(x)\phi_s(x_-)$$

$$\lambda^0 \qquad\qquad \lambda^0 \qquad\qquad \lambda^2$$

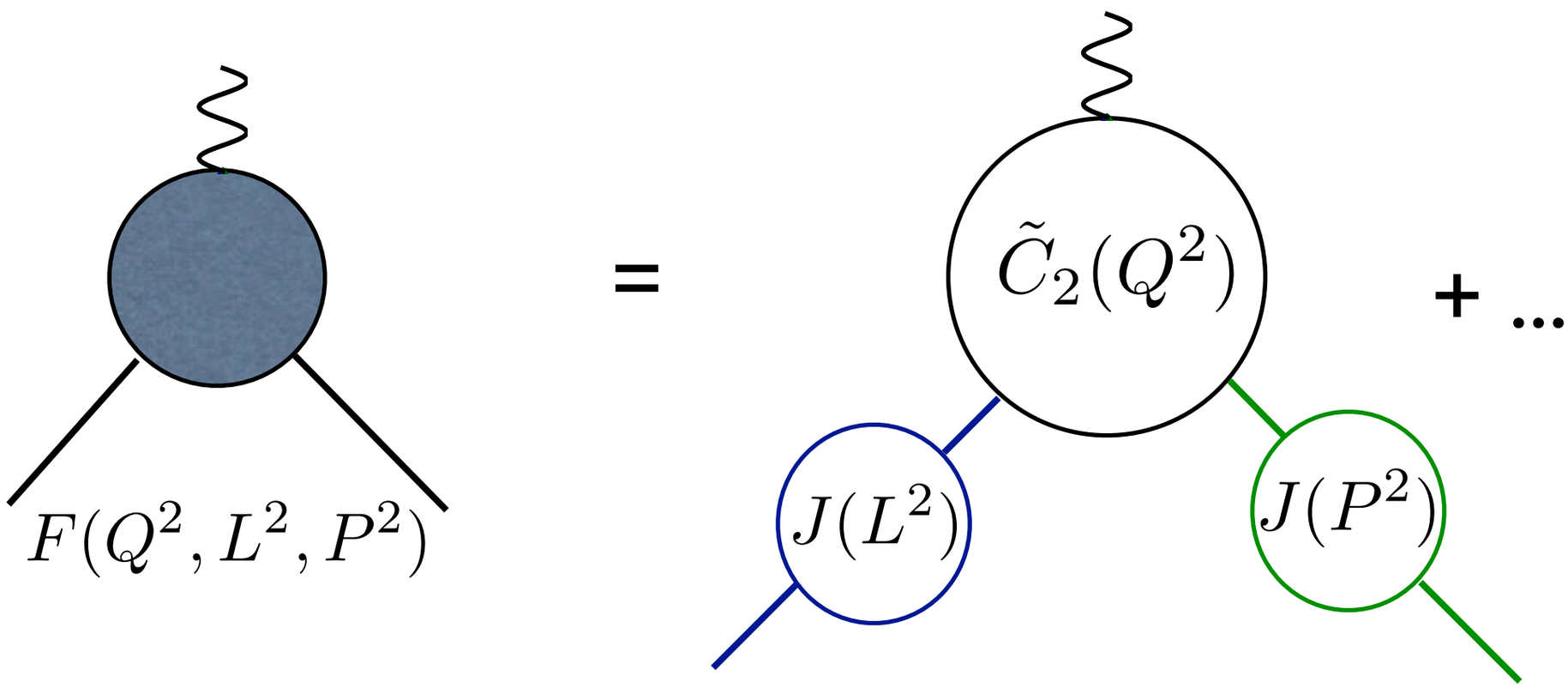
Soft-collinear interactions are power suppressed.

Also, the current operators with more fields are power suppressed. At leading power

$$J(x) = J_2(x) = \int ds\,dt\,C_2(s,t)\,\phi_{c1}(x+s\bar{n})\phi_{c2}(x+tn)$$

Factorization of the Sudakov form factor in d=6

The structure of the effective theory implies, that at leading power in λ, and to all orders in perturbation theory:



Our first factorization theorem!