Effective Field Theory Serie 3

Euler–Heisenberg Lagrangian 23. 4. 2024

In this exercise we study the Euler–Heisenberg Lagrangian,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{C_1}{m_e^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{C_2}{m_e^4} F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} + \mathcal{O}\left(\frac{E^6}{m_e^6}\right),$$

in great detail, including the matching to QED.

1. Derive the momentum-space Feynman rules for the interaction terms, symmetrizing in the external legs.

2. Using power-counting arguments, we showed in the lecture that the $\gamma\gamma \to \gamma\gamma$ cross section is suppressed by $\alpha^4 E^6/m_e^8$ at low photon energies. What is the relevant suppression factor for the $\gamma\gamma \to \gamma\gamma\gamma\gamma$ process?

3. Using the above Feynman rules, compute the center-of-mass cross section

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{1}{4\pi^2} (48 C_1^2 + 40 C_1 C_2 + 11 C_2^2) \frac{E^6}{m_e^8} \times (3 + \cos^2\theta)^2.$$

4. Compute the matching for the coefficients C_1 and C_2 along the lines sketched in the lecture. Using the result $C_1 = -\alpha^2/36$ and $C_2 = 7\alpha^2/90$, you should recover the known cross section

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = 139 \Big(\frac{\alpha^2}{180\pi}\Big)^2 \big(3 + \cos^2\theta\big)^2 \frac{E^6}{m_e^8}.$$