Effective Field Theory Serie 2

Continuum EFT 19. 3. 2023

1. Consider the following theory (in $d = 4$) with a heavy scalar ϕ_H and a light scalar ϕ_L:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_L \, \partial^\mu \phi_L - \frac{m^2}{2}\phi_L^2 + \frac{1}{2}\partial_\mu \phi_H \, \partial^\mu \phi_H - \frac{M^2}{2}\phi_H^2 - \frac{\lambda_L}{4!}\phi_L^4 - \frac{\lambda_{HL}}{4}\phi_L^2\phi_H^2 - \frac{\lambda_H}{4!}\phi_H^4 - \frac{g}{2}\phi_H\phi_L^2.$$

At low energies $E \sim m \ll M$, the physics is described by an effective Lagrangian that depends only on the light field and has the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu \phi_L \, \partial^\mu \phi_L - \frac{\tilde{m}^2}{2}\phi_L^2 - \frac{\tilde{\lambda}}{4!}\phi_L^4 - \frac{C_{2,4}}{2M^2}\phi_L \, \Box^2 \, \phi_L - \frac{C_{4,2}}{4!M^2}\,\phi_L^2 \, \Box \phi_L^2 - \frac{C_{6,0}}{6!M^2}\phi_L^6,$$

up to terms suppressed M^4.

a.) Show that the above form of the Lagrangian $\mathcal{L}_{\text{eff}}$ is indeed the most general one up to dimension 6. Remember that one can use integration by parts to eliminate terms from the action.

b.) Perform a field redefinition in the effective theory

$$\phi_L(x) \to \phi_L(x) + \frac{\alpha}{M^2}\,\Box \phi_L(x) + \frac{\beta}{M^2}\,\phi_L^3(x),$$

dropping any $1/M^4$ terms. Show that a suitable choice of α and β eliminates the two operators $\phi_L \, \Box^2 \, \phi_L$ and $\phi_L^2 \, \Box \, \phi_L^2$ from $\mathcal{L}_{\text{eff}}$.

c.) Draw the diagrams contributing to the two-point function up to one loop in the full and the effective theory. Use a double line to denote the heavy field and a single line for the light field.

d.) Draw the diagrams contributing to the four-point function at one loop in the full and the effective theory. One representative of each topology is enough (additional diagrams, related by the exchange of external legs, need not be drawn).

2. Consider QED in the vacuum sector ($N_{e^-} + N_{e^+} = 0$) at low energies $E_\gamma \ll m_e$. In this case, electrons and positrons only arise as virtual particles and can be integrated out, i.e., one can write down an effective Lagrangian $\mathcal{L}_{\text{eff}}(A_\mu)$ that contains only the photon field A_μ.

a.) Write down the most general (gauge invariant!) Lagrangian $\mathcal{L}_{\text{eff}}(A_\mu)$ including operators up to dimension 4.

b.) Write down the (minimal but complete) Lagrangian $\mathcal{L}_{\text{eff}}(A_\mu)$ including operators up to dimension 6.

3. In Exercise 1a.), we have shown that due to integration-by-parts identities there is only a single operator with four powers of the scalar field ϕ_L and two derivatives. In this exercise, we generalize this statement for operators additional powers of the scalar field and/or additional derivatives.

   a.) Show that there is always only a single independent operator for two fields and an arbitrary number of derivatives.

   b.) Show that there is only a single operator with two derivatives and arbitrary powers of the scalar field.

   c.) Show that starting with four derivatives and four or more fields, there are several independent operators.