1. Consider then the operators

$$O_1 = \phi^6(x) \qquad O_2 = \bar{\psi}(x)\psi(x)\,, \qquad O_3 = \phi(x)\,\bar{\psi}(x)\psi(x)\,,$$
$$O_4 = \bar{\psi}(x)\psi(x)\,\bar{\psi}(x)\psi(x) \qquad O_5 = F_{\mu\nu}F^{\mu\nu} \qquad O_6 = \bar{\psi}(x)\,\sigma_{\mu\nu}F^{\mu\nu}\,\psi(x)\,,$$

   and determine their operator dimension in $d$ space-time dimensions.

   To do so, derive first the mass dimensions of the scalar fields $\phi$, spin-1/2 fields $\psi$, and vector field $A_\mu(x)$. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. by requiring that the free action be dimensionless (work in $d$ dimensions).

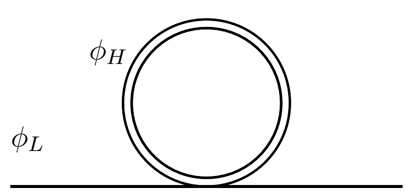
   Determine whether the operators $O_i$ are relevant, marginal, or irrelevant in $d = 2$, 3 and 4.

2. Compute the surface $\Omega_d$ of the $d$-dimensional unit-sphere. To do so, it is useful to consider the integral

$$\left[\int_{-\infty}^{\infty} dk\, e^{-ak^2}\right]^d = \int_{-\infty}^{\infty} d^dk\, e^{-a\mathbf{k}^2} = \Omega_d \int_0^{\infty} dk\, k^{d-1}\, e^{-ak^2}\,.$$

   After computing the integral over the length $k = |\mathbf{k}|$ one can solve for $\Omega_d$. Check that you reproduce the known results for $d = 1, 2, 3$ and confirm the value for $d = 4$ used in the lecture. How does the surface behave when the dimension becomes large?

3. Consider scalar $\phi^4$ field theory in the Wilsonian framework, as we did in the lecture. Compute the one-loop self-energy correction from integrating out the high-energy part $\phi_H$ of the field:




   This contribution modifies the mass term in the low-energy theory. By how much does the mass change if we integrate out the modes $\phi_H$ with momentum $b\Lambda < |k| < \Lambda$?