

Solutions EFT, Serie 3

1.), 3.) see Mathematica notebook / PDF

feynman-rule-Euler-Heisenberg.nb

4.) notebook

Euler-Heisenberg-Matching.nb

2.) let's first consider $\gamma\gamma \rightarrow \gamma\gamma$

$$\sigma \sim \left| \begin{array}{c} \text{diagram} \\ \uparrow \\ \frac{C_{1,2}}{m_e^4} \end{array} \right|^2 \sim \frac{\alpha^4}{m_e^8} \cdot E_\gamma^n \cdot f(\theta)$$

scattering angle
↓

Dimension of σ is area, so $\sigma \sim \frac{1}{E^2}$

dimensional analysis yields $n = 6$

$$\sigma = \alpha^4 \frac{E_\gamma^6}{m_e^8} f(\theta)$$

The explicit calculations in 3.) & 4.)

yield $f(\theta) = 13g \left(\frac{1}{180\pi} \right)^2 (3 + \cos^2 \theta)^2$

Diagrams for $\gamma\gamma \rightarrow \gamma\gamma\gamma\gamma$ include

$\sigma \sim \left| \text{diagram} + \dots \right|^2$

$\sim \left(\frac{1}{m^8} \right)^2 E_\gamma^n \sim \alpha^8 \frac{E_\gamma^{14}}{m^{16}} \quad (A)$

but perhaps we can use a higher-order

Lagrangian to directly produce $\gamma\gamma \rightarrow \gamma\gamma\gamma\gamma$?

$\sigma \sim \left| \text{diagram} + \dots \right|^2 \quad (B)$

↑
needs 6 γ 's $\hat{=} F^6$

↙ dimension 12

e.g. $\Delta\mathcal{L} = \frac{C_6}{m_c^8} (F_{\mu\nu} F^{\mu\nu})^3$

we see that these contributions enter at the same order as the ones in category (A).

let us also count power of α



The diagram shows two Feynman diagrams separated by an equals sign. The left diagram is a vertex correction: a central vertex with four external wavy lines, and a loop of two wavy lines attached to the vertex. The right diagram is a self-energy loop: a central vertex with two external wavy lines, and a loop of two wavy lines attached to the vertex. To the right of the second diagram is the expression $\sim \alpha^3$.

→ This is less suppressed than (A)!

$$\rightarrow \sigma \sim \alpha^6 \frac{E^4}{m_e^{16}}$$