Solutions EFT, Serie 3

1.), 3.) see Mathematica notebook / PDF

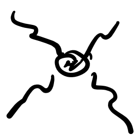
feynman-rule_Euler_Heisenberg.nb

4.) notebook

Euler-Heisenberg-Matching.nb

2.) let's first consider $\gamma\gamma \to \gamma\gamma$

scattering angle

$\sigma \sim \Big|$  $\Big|^2 \sim \frac{\alpha^4}{m_e^8} \cdot E_\gamma^n \cdot f(\theta)$

$\frac{C_{1,2}}{m_e^4}$

Dimension of σ is area, so $\sigma \sim \frac{1}{E^2}$

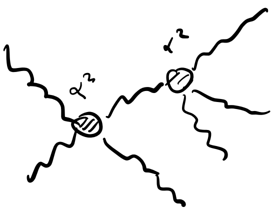
dimensional analysis yields $n = 6$

$$\sigma = \alpha^4 \frac{E_\gamma^6}{m_e^8} f(\theta)$$

The explicit calculations in 3.) & 4.)

yield $\quad f(\theta) = 139\left(\frac{1}{180\pi}\right)^2 \left(3+\cos^2\theta\right)^2$

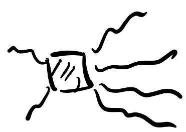
Diagrams for $\gamma\gamma \longrightarrow \gamma\gamma\gamma\gamma$ include

$\sigma \sim \Big|$ 

$+ \cdots \Big|^2$

$$\sim \left(\frac{1}{m^8}\right)^2 E_\gamma^n \sim \alpha^8 \frac{E_\gamma^{14}}{m^{16}} \qquad \text{(A)}$$

but perhaps we can use a higher-order Lagrangian to directly produce $\gamma\gamma \to \gamma\gamma\gamma\gamma$?

$\sigma \sim \Big|$  $+ \cdots \Big|^2 \qquad$ (B)

$\uparrow$

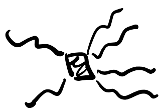
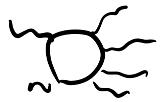
needs 6 γ's $\hat{=}\ F^6$

dimension 12

$$\text{e.g.} \quad \Delta\mathcal{L} = \frac{C_6}{m_e^8}\left(F_{\mu\nu}F^{\mu\nu}\right)^3$$

We see that these contributions enter at the same order as the ones in category (A).

Let us also count powers of α

 $=$  $\sim \alpha^3$

$\rightarrow$ This is less suppressed than (A)!

$$\longrightarrow \quad \sigma \sim \alpha^6 \frac{E_\gamma^{14}}{m_e^{16}}$$