Exercise 1 : Wilsonian EFT

1.) Action is dimensionless. From kinetic terms one reads off

$$\int d^d x \, (\partial_\mu \phi)^2 \longrightarrow [\phi] = \frac{d-2}{2}$$

$$\int d^d x \, \bar{\Psi} \not{\partial} \Psi \longrightarrow [\Psi] = \frac{d-1}{2}$$

$$\int d^d x \, (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \longrightarrow [A_\mu] = \frac{d-2}{2}$$

Note: $[D_\mu] = [\partial_\mu - ieA_\mu] = 1$

$$\rightarrow [e] = \frac{4-d}{2}$$

operator dimension:

O	d	d = 2	d = 3	d = 4
1	$3(d-2)$	0	3	6
2	$d-1$	1	2	3
3	$\frac{3d}{2}-2$	1	$\frac{5}{2}$	4
4	$2d-2$	2	4	6
5	d	2	3	4
6	$\frac{3d}{2}-1$	2	$7/2$	5

irrelevant relevant marginal

2.) Carrying out the integrals in the exercise, we get

$$\left(\int_{-\infty}^{\infty} dk \, e^{-ak^2} \right)^d = \left(\sqrt{\frac{\pi}{a}} \right)^d$$

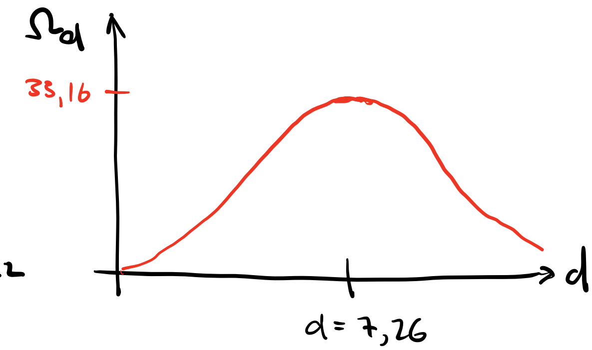
$$\int_0^{\infty} dk \, k^{d-1} e^{-ak^2} = \left(\frac{1}{\sqrt{a}} \right)^d \frac{1}{2} \underbrace{\int_0^{\infty} dt \, t^{d/2 - 1} e^{-t}}_{= \Gamma(d/2)}$$

$ak^2 = t$

$2ak\,dk = dt$

$$\rightarrow \left(\sqrt{\pi}\right)^d = \Omega_d \cdot \frac{1}{2} \Gamma\left(\frac{d}{2}\right)$$

$$\rightarrow \Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

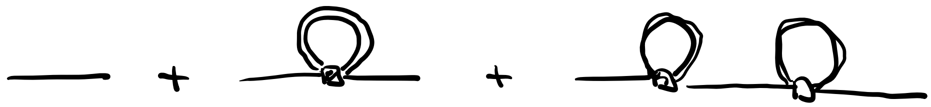


$\Omega_1 = 2$, $\Omega_2 = 2\pi$, $\Omega_3 = 4\pi$, $\Omega_4 = 2\pi^2$

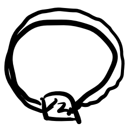
3.) we compute Γ_2, see (3.6) in script.

Let us first discuss how to obtain it.

The two-point function is given by

$G(p) =$  $+ \cdots$

Let's define Σ as:

$-\Sigma =$ 

Then

$$G(p) = \text{---} + \text{---}(-\Sigma)\text{---} + \text{---}(-\Sigma)\text{---}(-\Sigma)\text{---}$$

$$= \frac{1}{p^2+m^2} + \frac{1}{p^2+m^2}(-\Sigma)\frac{1}{p^2+m^2} + \cdots$$

$$= \frac{1}{p^2+m^2+\Sigma}$$

Truncated Green's function

$$\rightarrow \Gamma(p^2) = G^{-1}(p) \cdot G(p) \cdot G^{-1}(p) = G(p)$$

$$= p^2 + m^2 + \Sigma$$

To compute the diagram we need the vertex

$- S_E$

$$\lambda \int d^4x \, \frac{\phi_L^2 \phi_H^2}{2!\,2!} \longrightarrow \text{Feynman rule} : -\lambda$$

$$-\Sigma = $$  $$= -\frac{\lambda}{2} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2+m^2}$$

symm factor

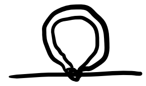
$$\Theta(|k| > \Lambda b)\, \Theta(|k| < \Lambda)$$

$d=4$

$$= -\frac{\lambda}{2} \frac{2\pi^2}{(2\pi)^4} \int_{\Lambda b}^{\Lambda} dk \, k^3 \cdot \frac{1}{k^2+m^2}$$

$$= -\frac{\lambda}{16\pi^2} \int_{\Lambda b}^{\Lambda} dk \cdot k\left(1 + O\left(m^2/k^2\right)\right) = -\frac{\lambda}{32\pi^2} \Lambda^2 (1-b^2)$$

The full two point function is

$$\Gamma_2(p) = p^2 + m^2 - $$ 

$$= p^2 + \underbrace{m^2 + \frac{\lambda \Lambda^2}{32\pi^2}(1-b^2)}_{m^2(b)}$$

change in mass $\propto \Lambda^2$

Note that the loop does not change the coefficient of the p^2-term. A contribution $c(b) \cdot p^2$ would change the norm of the field. To then have canonical normalization of the kinetic term, one would renormalize the field.

For the quadratic action in Section 2.2.1.

we have rescaled

$$k \to b k' \quad ; \quad x \to x'/b$$

and $\tilde{\phi}(k) \to \tilde{\phi}'(k') \cdot b^{-\frac{d+2}{2}}$ to get

back an action with the original cutoff. We

can do the same here, which would again

lead to $m^2 \to m^2/b$, see (2.35).