

# Effective Field Theories

<http://www.becher.itp.unibe.ch/eft24/>

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# Tools for QFT computations

Expansion  
in the interaction  
strength:  
perturbation  
theory

Expansion in  
scale ratios:  
Effective Field  
Theories

Numerical  
methods:  
lattice simulations

Toy models:  
solvable models  
SUSY theories  
AdS/CFT

# Benefits of EFTs

- **Expansion** in scale ratios simplifies computations
- **Factorization** of physics at different energy scales
  - Separate perturbative from non-perturbative physics
  - Dimensional analysis
  - Perturbation theory works. (It cannot be applied in multi-scale problems due to large logarithms.)
- **Symmetries**
  - emergent: heavy-quark symmetry
  - approximate: chiral symmetry
- **General framework** also for cases where the full theory is not known, or cannot be used for computations

# Wilsonian vs. Continuum EFT

- **Wilsonian EFT** integrates out high-energy physics above some cutoff exactly, using path integral
  - top down approach, difficult in practice
  - works with hard cutoffs
  - provides physical picture of renormalization
- **Continuum EFT**
  - write down the most general low energy  $\mathbf{L}_{\text{eff}}$
  - determine coefficients of terms in  $\mathbf{L}_{\text{eff}}$  by matching
  - usually: dimensional regularization instead of hard cutoff

# Traditional low energy EFTs

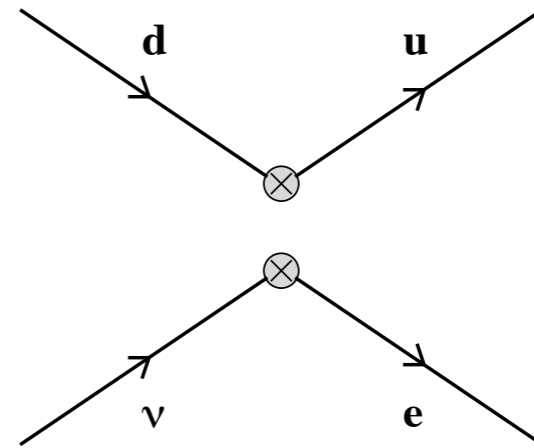
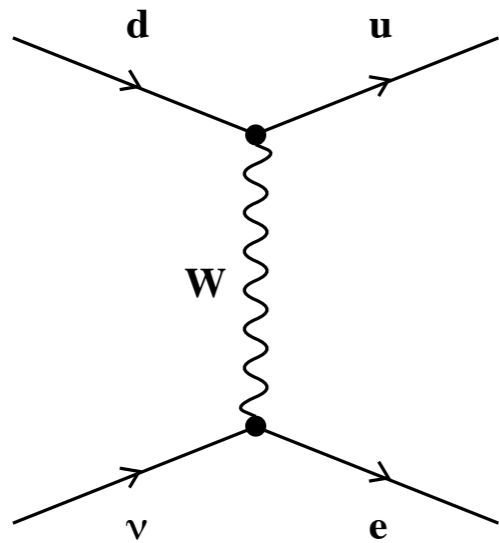
The effective theory is a standard relativistic quantum field theory, but includes non-renormalizable operators.

- higher-dim operators suppressed at low energies
- renormalizable up to a given power

Typically, such EFTs are obtained after integrating out heavy particles.

- Effective Lagrangian depends on light fields
- Higher-dim operators are induced from integrating out heavy degrees of freedom

# Fermi theory



$$\frac{1}{p^2 - M_W^2}$$



$$\frac{1}{-M_W^2} \left( 1 + \frac{p^2}{M_W^2} + \dots \right)$$

# A tower of EFTs

effective theory	full theory	energy scale $\Lambda$	fields in $\mathcal{L}_{\text{eff}}$
Fermi theory (QED+QCD+eff. weak)	SM	$M_W \approx 80 \text{ GeV}$	$l, \nu, q, g, \gamma$

# A tower of EFTs

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Fermi theory (QED+QCD+eff. weak)	SM	$M_W \approx 80 \text{ GeV}$	$l, \nu, q, g, \gamma$
CHPT + QED	QCD + QED + eff. weak	$m_\rho \approx 1 \text{ GeV}$	$\pi, K, \gamma, \dots$



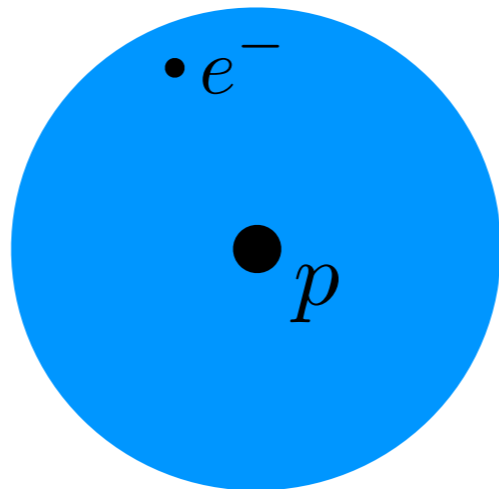
# A tower of EFTs

effective theory	full theory	energy scale $\Lambda$	fields in $L_{\text{eff}}$
SM	?	$\Lambda_{\text{new}} = ?$	$l, \nu, q, g, \gamma, H, W, Z$
Fermi theory (QED+QCD+eff. weak)	SM	$M_W \approx 80 \text{ GeV}$	$l, \nu, q, g, \gamma$
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# Modern EFTs: e.g. NRQED

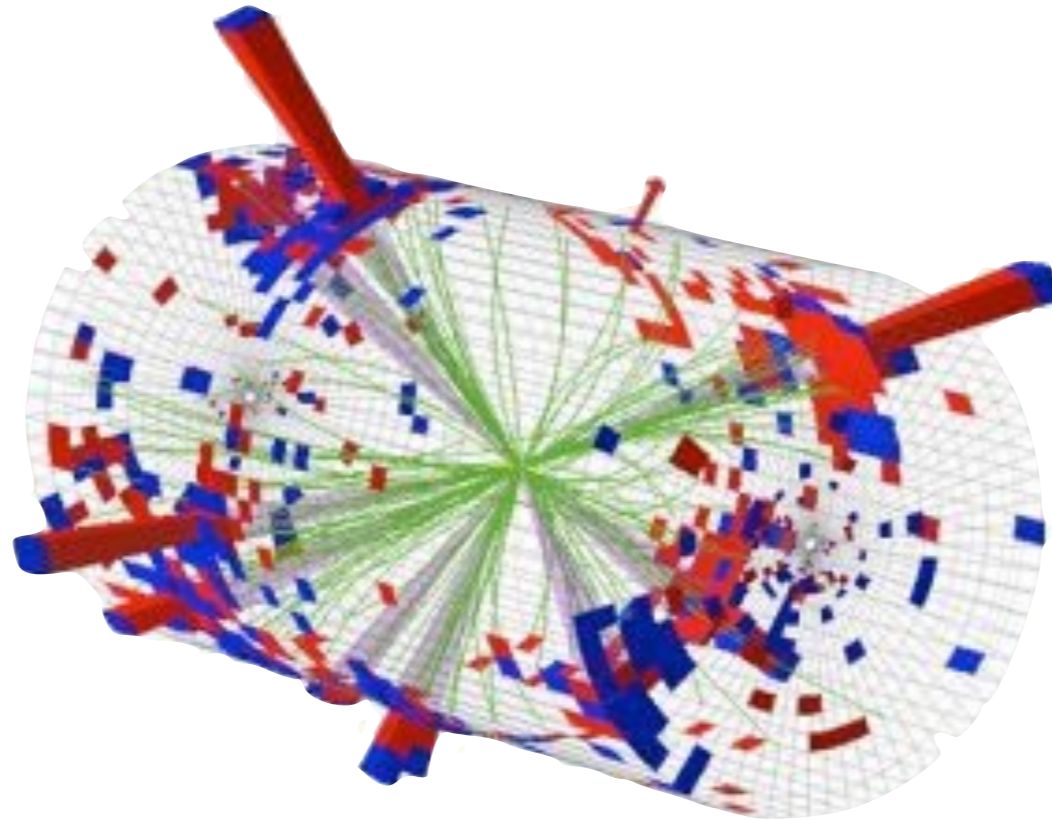
Many examples of scale hierarchies in QFT in which the low-energy part does not consist of light particles with low momentum.

e.g. non-relativistic problems  $E_{\text{kin}} \ll |\vec{p}| \ll m$



- Cannot simply integrate out  $e^-$  or  $p$ .
- Different components of the momentum scale differently

# Jet physics at the LHC



Many scale hierarchies!

$$\sqrt{s} \gg p_{\text{Jet}}^T \gg M_{\text{Jet}} \gg E_{\text{out}} \gg m_{\text{proton}} \sim \Lambda_{\text{QCD}}$$

→ Soft-Collinear Effective Theory (SCET)

# Modern complications

- EFT is tailored to physics at hand: reference vectors,...
- Particles may be described by several fields (modes)
  - Two kinds of photon fields in NRQED: soft and ultra-soft
  - Various types of soft and collinear particles in SCET
- Not all momentum components are small: non-localities along the directions of large momenta
  - parton-distribution functions
  - Coulomb potential

# Outline

## 1 Introduction

## 2 The Wilsonian effective action [1, 2]

- 2.1 Integrating out high-energy modes . . . . .
- 2.2 Classification of operators . . . . .
- 2.3 Renormalization group . . . . .

## 3 Continuum effective theory [3–5]

- 3.1 Tree-level matching calculations . . . . .
- 3.2 Field redefinitions . . . . .
- 3.3 Matching at higher orders . . . . .
- 3.4 Power counting . . . . .
- 3.5 Renormalization group improved perturbation theory . . . . .

## 4 The Standard Model at low energies

- 4.1 Euler Heisenberg Theory [6] . . . . .
- 4.2 Decoupling of heavy flavors . . . . .
- 4.3 Effective weak Hamiltonian (Fermi Theory) [7, 8] . . . . .
- 4.4 Chiral Perturbation Theory [9–11] . . . . .

<b>5</b>	<b>Heavy quarks and non-relativistic systems</b>	
5.1	Heavy Quark Effective Theory (HQET) [8, 12]	.....
5.2	Nonrelativistic Effective Theories [4, 13]	.....
5.2.1	Nonrelativistic QCD (NRQCD)	.....
5.2.2	Potential Nonrelativistic QCD (pNRQCD) [14–16]	.....
<b>6</b>	<b>Energetic particles and jet physics</b>	
6.1	Method of regions [17]	.....
6.2	Soft-Collinear Effective Theory [18, 19]	.....
<b>7</b>	<b>Further examples</b>	
7.1	Fermi liquids and the BCS theory of superconductivity [1, 20]	.....
7.2	The Standard Model of Particle Physics [3, 23]	.....
7.3	Nucleon-nucleon interaction EFTs [20]	.....
7.4	Symanzik Effective Theory [21, 22]	.....
7.5	General relativity for extended objects [24]	.....
7.6	Thermal effective field theories[25]	.....
7.6.1	Dimensionally reduced effective field theory for hot QCD	.....
7.6.2	Hard Thermal Loops effective theory	.....