B. Feynman rules for derivative couplings

The Feynman rules are obtained by Fourier transforming and symmetrizing:

E.g. Consider $\delta \mathcal{L} = \frac{g}{4!}\, \phi^2(x)\, \Box\, \phi^2(x)$

Write:
$$\phi(x) = \int_k e^{-ikx}\, \tilde{\phi}(k) \equiv \int \frac{d^dk}{(2\pi)^d}\, e^{-ikx}\, \tilde{\phi}(k)$$

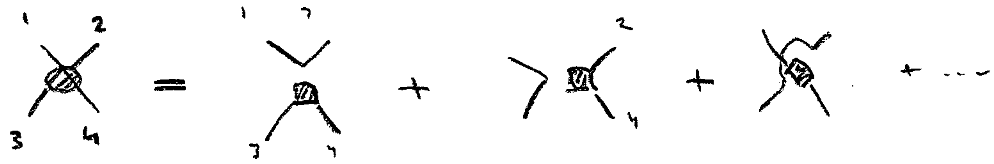
k corresponds to an incoming momentum.

Fourier transform:

$$\int d^dx\, \delta\mathcal{L} = \int_{k_1}\int_{k_2}\int_{k_3}\int_{k_4} \int d^dx\, \frac{g}{4!}\, \tilde{\phi}(k_1)\tilde{\phi}(k_2)\left[-(k_3+k_4)^2\right] \tilde{\phi}(k_3)\tilde{\phi}(k_4)\, e^{-i(k_1+k_2+k_3+k_4)x}$$

$$= -\frac{g}{4!} \int_{k_1}\int_{k_2}\int_{k_3}\int_{k_4} \tilde{\phi}(k_1)\tilde{\phi}(k_2)(k_3+k_4)^2\,\tilde{\phi}(k_3)\tilde{\phi}(k_4)\, (2\pi)^d\, \delta(k_1+k_2+k_3+k_4)$$

In this form, we would need to remember on which fields the derivatives act, so



It is easier to always symmetrize to avoid this.

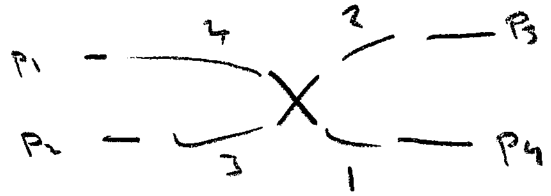
Because of momentum conservation $(k_3+k_4)^2 = (k_1+k_2)^2$, etc. So there are only three distinct momentum pairs.

$$\int d^dx\, \delta\mathcal{L} = -\frac{g}{4!} \int_{k_1}\int_{k_2}\int_{k_3}\int_{k_4} \frac{1}{3}\left[(k_1+k_2)^2 + (k_1+k_3)^2 + (k_1+k_4)^2\right]$$

$$* (2\pi)^d\, \delta(k_1+k_2+k_3+k_4)\, \tilde{\phi}(k_1)\tilde{\phi}(k_2)\tilde{\phi}(k_3)\tilde{\phi}(k_4)$$

is now completely symmetric.

Let's compute the 4-point function. As in ϕ^4 theory, there are 4! identical



contractions, which cancel the $\frac{1}{4!}$ prefactor. For the remainder, we get

$$T^{amp} = -ig\,\frac{1}{3}\left[(p_1+p_2)^2 + (p_1-p_3)^2 + (p_1-p_4)^2\right]$$

where I have assumed that p_1 and p_2 are incoming and p_3, p_4 outgoing. On the mass shell $p_i^2 = m^2$, one has

$$T^{amp} = -ig\,\frac{1}{3}\, 4m^2 .$$