4.2.2 Heavy flavors in QCD

The quark masses are very hierarchical and for many application one will need to integrate out the heavy flavors $m_t \approx 173\,\text{GeV}$ and $m_b = 5\,\text{GeV}$. The masses of these quarks are large enough that the matching can be performed perturbatively.

(For the charm $m_c = 1{,}3\,\text{GeV}$, $\alpha_s(m_c) = 0{,}34$, this is also true, but the corrections will be significant.)

Let us first discuss $\mathcal{L}_{eff}$ up to dimension 6 and then the matching for the $d=4$ Lagrangian.

All the operators found in QED are also allowed in QCD. The effective QCD Lagrangian after integrating out the top quark has the form

$$\mathcal{L}_{d=4} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} - \sum_{f=1}^{5} \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

This is simply QED with 5 flavors!

For $d=6$, we have operators

$$O^{ff'}_{(si)} = \bar{\psi}_f \gamma^{[\mu_1} \cdots \gamma^{\mu_i]} \psi_f \, \bar{\psi}_{f'} \gamma_{[\mu_1} \cdots \gamma_{\mu_i]} \psi_{f'}$$

$$O^{ff'}_{(oi)} = \bar{\psi}_f t^a \gamma^{[\mu_1} \cdots \gamma^{\mu_i]} \psi_f \, \bar{\psi}_{f'} t^a \gamma^{[\mu_1} \cdots \gamma^{\mu_i]} \psi_{f'}$$

odd numbers (even numbers suppressed by m_f)

Since QCD does not distinguish the flavors*, the Wilson coefficients of $O^{ff'}_{5i}$ are independent of the flavor indices, i.e. we only need the two operators

$$O_{(5i)} = \sum_{ff'} O^{ff'}_{(5i)} \; ; \quad O_{(0i)} = \sum_{ff'} O^{ff'}_{(0i)} \, .$$

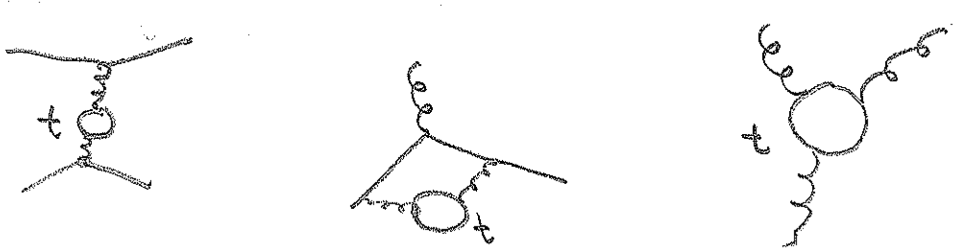
As in QED, we get

$$O_{mag} = \sum_f m_f \, \bar{\psi} \, \sigma^{\mu\nu} G_{\mu\nu} \psi$$

and there is one additional operator, namely

$$O_3 = f^{abc} \, F^{\mu,a}_{\ \nu} \, F^{\nu,b}_{\ \sigma} \, F^{\sigma,c}_{\ \mu} \, .$$

The leading contributions to the Wilson coefficients of $O_{(0i)}$ and O_{mag} and O_3 arise from



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* Except for the quark masses, which can be set to zero for the matching computation.

Let us now discuss the matching for $\mathcal{L}_{d=4}$, which contains as Wilson coefficients the coupling constant g_s and m_f. Let us discuss the issue for the coupling constant $\alpha_s(\mu) = \frac{g_s^2(\mu)}{4\pi}$.

We denote the coupling by $\alpha_s^{n_f}(\mu)$ to distinguish α_s in the theory with $n_f = 6$ from $\alpha_s^5(\mu)$ in the theory where the top is integrated out. Then we proceed as in our toy model. At a scale $\mu_m \approx m_t$ one derives a relation

$$\alpha_s^5(\mu_m) = \alpha_s^6(\mu_m)\, \zeta_A^{-1}\left[\alpha_s^6(\mu_m)\right]$$

The simplest way to obtain ξ_A is to compute the gluon propagator in both theories:

$$G_{\mu\nu} = \frac{i Z_A^{os}}{p^2} \left(-g_{\mu\nu} + \ldots \right)$$

Rescaling the coupling by ξ_A is the same as rescaling the gluon field. One can then show that

$$\underset{\swarrow \text{bare}}{\xi_A^{(0)}} = \frac{Z_A^{6}}{Z_A^{5}} \qquad \text{with } Z_A = \frac{1}{1-\Pi(0)}$$

If one chooses $\mu_m = m_t(\mu_m)$ the expression for ξ_A is especially simple

$$\xi_A(m_t) = 1 + \left(\frac{13}{3} C_F - \frac{32}{9} C_A \right) T_F \left(\frac{\alpha(m_t)}{4\pi} \right)^2$$

Note that the QCD coupling runs differently for $n_f = 5$ and $n_f = 6$:

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \beta(\alpha_s(\mu))$$

$$\beta(\alpha_s) = -2\alpha_s \left[\beta_0 \left(\frac{\alpha_s}{4\pi}\right)^2 + \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + \dots \right]$$

with $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} n_f T_F$
$\Uparrow$

This running and decoupling has been implemented into a computer code RunDec, see slides.