3.4 Power counting

In our matching calculation we have assumed that the higher order Lagrangians do not contribute at leading power. At the tree level, this is obvious, since the operators in the power suppressed Lagrangians have additional derivatives and or fields. But in loop integrals the momenta are large, so it's not trivial that higher derivative terms are suppressed.

E.g. the contribution of $\frac{1}{(M^2)^n}\phi_L^2 \Box^n \phi_L^2$ to the two point function at zero external momentum is

$$\delta\Sigma \propto \frac{1}{(M^2)^n}\int d^dk \, \frac{(k^2)^n}{(k^2-m^2)} = \,?$$

The beauty of dim. reg. is that the loop integrals in the EFT only depend on low-energy scales, therefore by dimensional analysis $\delta\Sigma \propto (m^2)^{d/2-1} \times (m^2)^n \cdot \frac{1}{(M^2)^n}$

So the loop graph is indeed suppressed by $\left(\frac{m^2}{M^2}\right)^n$.

Note that in cut-off regularization

$$\frac{1}{(M^2)^n}\int^{\Lambda} d^4k \; \frac{(k^2)^n}{k^2-m^2} \;\propto\; \Lambda^{d-2} \times \Lambda^{2n}/M^{2n} + \dots$$

the loop contributions of higher-dim operators are unsuppressed. The terms which violate the tree level power counting are trivial cut-off terms but they make computations cumbersome.

In contrast, the power counting in dim. reg is very simple. To get a quantity up to $\left(\frac{1}{M^2}\right)^n$ accuracy, we need the Lagrangian up to $\left(\frac{1}{M^2}\right)^n$ and the $\left(\frac{1}{M^2}\right)^n$ corrections arise from diagrams with:

a single vertex from $\mathcal{L}_{eff}^{(n)}$,

or one from $\mathcal{L}_{eff}^{(n-m)}$ and one from $\mathcal{L}^{(m)}$,

ore one from $\mathcal{L}_{eff}^{(n-m_1-m_2)}$ and one from $\mathcal{L}^{(m_1)}$ and $\mathcal{L}^{(m_2)}$,

etc.