EFFECTIVE FIELD THEORIES

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1. Introduction

An effective field theory (EFT) is a field theory which only describes the physics below some scale Λ, as opposed to a fundamental field theory which should be valid up to arbitrarily high energies.

What are examples of effective theories? Every field theory we know! No one in his right mind believes that the Standard Model of particle physics is valid up to arbitrarily high energies. For QCD, it would be consistent to assume that it is valid up to infinite energies (because of asymptotic freedom), while QED is only consistent up to extremely high but finite energies.

Once one admits that one's theory is only valid up to some scale Λ, the usual requirement of renormalizability is too strong. One only asks that a finite number of parameters describes the physics up to effects suppressed by $\left(\frac{E}{\Lambda}\right)^n$, where E is the energy of the particles in the effective theory. (the higher n, the more parameters will be needed).

Here are some examples of EFTs:

	low energy EFT	high-energy theory "full" theory	Fundamental scale Λ
I.)	SM	?	1 TeV ?
II.)	Fermi theory	SM	$m_W \sim 80$ GeV
III.)	CHPT (chiral pert. theory)	QCD / SM	$m_p \sim 1$ GeV

The tree examples display different aspects of why EFTs are useful.

I.) We don't know the corresponding full theory, but once we admit that the SM is an effective theory, we should search for low-energy $(E/\Lambda)^n$ effects ($n=1,2$) to find out at what scale new physics enters.

There is a large program of precision measurements looking for such effects:

precision EW suggests $\Lambda \gtrsim 5\,\text{TeV}$

precision flavor physics $\Lambda \gtrsim 1-100\,\text{TeV}$

Theoretical arguments (see later) suggest $\Lambda \sim 200\,\text{GeV}$, the fact that the explicit bounds are higher are known as the "little hierarchy problem" and the "flavor problem".

II.) Fermi theory. Here we know the full theory (the SM) and we can derive the couplings in the EFT from full-theory calculations. Nevertheless, the EFT is useful: calculations become much simpler and perturbation theory is much better behaved.

III.) In CHPT, the full theory is so complicated that we cannot calculate the couplings constants in $\mathcal{L}_{eff}$ from QCD. (With lattice QCD, this becomes now to some extent possible).

Nevertheless, the EFT is very useful because it implements the symmetries of the full theory. They imply that at very low energies, you can describe all scattering processes of pions in terms of F_π and the meson masses.

Because of the complexity of QCD, many different EFTs, taylored to various physics situations have been developed. Most QCD calculations (including lattice calculations) rely on the use of EFTs.

In the following, we will first discuss the Wilsonian effective action. This is obtained by explicitly integrating out all physics above a cut-off Λ. It will allow us to discuss many of the conceptual issues, but having an explicit cut-off makes computations very cumbersome.

In practice, it is much easier to use "continuum effective theory", i.e. to work with dimensional regularization in the $\overline{MS}$ scheme.

We'll develop the necessary formalism using a toy theory, in which a heavy scalar field is integrated out.

After this we look at EFTs for different sectors of the SM and apply them to a variety of physical problems.