One-loop matching for Euler-Heisenberg EFT

Thomas Becher (thomas.becher@unibe.ch), April 2024

```
<< DiracAlgebra`
```

The DiracAlgebra package is not public but can be obtained from the author.

Kinematics

```
In[2]:= kinematics = {p1 · p1 → 0, p2 · p2 → 0, p3 · p3 → 0,
           p4 · p4 → 0, p1 · p2 → s/2};

In[3]:= onShell = Take[kinematics, 4]

Out[3]= {p1 · p1 → 0, p2 · p2 → 0, p3 · p3 → 0, p4 · p4 → 0}
```

Loop integral

General form, from appendix A. We suppress a factor

```
In[4]:= preFactor = i/(4 π)^(d/2);
```

which will be added later. The remaining integral reads

```
In[5]:= loopInt2[α_, β_, m2_] :=
          (-1)^(α+β) (m2)^(α-β+d/2)
            Gamma[d/2 + α] Gamma[β - α - d/2] / (Gamma[d/2] Gamma[β]) /. d → 4 - 2 ϵ //
          Simplify;
```

We only need the integral for $\alpha = 0$!

In[6]:=
```
tadpole[n_] := loopInt2[0, n, m^2]
```

Count external momenta as small

The loop momentum k is large!

In[7]:=
```
count = {k · a_ :> λ k · a /; a =!= k,
   (a_) · (b_) :> λ^2 a · b /; (a =!= k) ∧ (b =!= k),
   v_a_ :> λ v_a /; v =!= k};
```

Box integral B

In[8]:=
```
denomB =
```
$$\frac{1}{k\cdot k - m^2}\,\frac{1}{(k+p2)\cdot(k+p2) - m^2}\,\frac{1}{(k+p3-p1)\cdot(k+p3-p1) - m^2}\,\frac{1}{(k-p1)\cdot(k-p1) - m^2}$$
```
  // γExp;
```

In[9]:= `denomB2 = denomB /. onShell /. count`

Out[9]= $1 \big/ \left(\left(-m^2 + k \cdot k \right) \left(-m^2 + k \cdot k - 2\,\lambda\, k \cdot p1 \right) \left(-m^2 + k \cdot k + 2\,\lambda\, k \cdot p2 \right) \left(-m^2 + k \cdot k - 2\,\lambda\, k \cdot p1 + 2\,\lambda\, k \cdot p3 - 2\,\lambda^2\, p1 \cdot p3 \right) \right)$

We suppress a factor $(i\, e)^4$ from vertex and i^4 from propagators, as well as a factor (-1) from the closed fermion loop. These will be added at the end.

In[10]:= `numeratorB0 = γμ1 ∧ (m 𝕀 + k̂) ∧ γμ2 ∧ (m 𝕀 + k̂ + p̂2) ∧ γμ4 ∧ (m 𝕀 + k̂ - p̂1 + p̂3) ∧ γμ3 ∧ (m 𝕀 + k̂ - p̂1)`

Out[10]= $\gamma_{\mu 1} \wedge \left(m\, \mathbb{I} + \hat{k} \right) \wedge \gamma_{\mu 2} \wedge \left(m\, \mathbb{I} + \hat{k} + \hat{p2} \right) \wedge \gamma_{\mu 4} \wedge \left(m\, \mathbb{I} + \hat{k} - \hat{p1} + \hat{p3} \right) \wedge \gamma_{\mu 3} \wedge \left(m\, \mathbb{I} + \hat{k} - \hat{p1} \right)$

In[11]:= `numeratorB0 /. {p3 → p1, p4 → p2}`

Out[11]= $\gamma_{\mu 1} \wedge \left(m\, \mathbb{I} + \hat{k} \right) \wedge \gamma_{\mu 2} \wedge \left(m\, \mathbb{I} + \hat{k} + \hat{p2} \right) \wedge \gamma_{\mu 4} \wedge \left(m\, \mathbb{I} + \hat{k} \right) \wedge \gamma_{\mu 3} \wedge \left(m\, \mathbb{I} + \hat{k} - \hat{p1} \right)$

In[12]:= `numeratorB1 = numeratorB0 // γTrace;`

In[13]:= `numeratorB2 = numeratorB1 /. count /. onShell;`

Construct full amplitude

In[14]:=
```
ampFull = 2 one denomB2 numeratorB2 +
    2 two
     (denomB2 numeratorB2 /.
       {μ3 → μ4, μ4 → μ3, p3 → p4, p4 → p3}) +
    2 three
     (denomB2 numeratorB2 /.
       {μ2 → μ4, μ4 → μ2, p2 → -p4, p4 → -p2});
```

Note: The labels `one`, `two`, `three` can be set to 1 at the end. They are only used to illustrate cancellations between the diagrams.

In[15]:=
```
ampForward = ampFull /. {p3 → p1, p4 → p2};
```

Contract indices

In[16]:=
```
projector1 = g_{μ1,μ2} g_{μ3,μ4};
```

In[17]:=
```
projector2 = g_{μ1,μ3} g_{μ2,μ4};
```

In[18]:=
```
structure1 = (projector1 ampForward // γSimp) /.
    onShell;
```

In[19]:=
```
structure1b = Series[structure1, {λ, 0, 4}] // Normal;
```

In[20]:=
```
structure1c = Collect[structure1b /. k·k → Δ + m^2,
    λ, Simplify];
```

In[21]:=
```
structure2 = (projector2 ampForward // γSimp) /.
    onShell;
```

In[22]:=
```
structure2b = Series[structure2, {λ, 0, 4}] // Normal;
```

In[23]:=
```
structure2c = Collect[structure2b /. k·k → Δ + m^2,
    λ, Simplify];
```

Now look at the terms order by order: structure 1

In[24]:=
```
orderZero = Collect[Coefficient[structure1c, λ, 0],
   {λ, Δ}, Simplify]
```

Out[24]=
$$\frac{64\,m^4\,(\text{one}+\text{three}+\text{two})}{\Delta^4}-\frac{32\,m^2\,((-3+d)\,\text{one}-2\,\text{three}+(-3+d)\,\text{two})}{\Delta^3}+\frac{8\,(-2+d)\,((-2+d)\,\text{one}-2\,\text{two}+d\,(-\text{three}+\text{two}))}{\Delta^2}$$

In[25]:=
```
orderZero2 = orderZero /. d → 4 - 2 ϵ /. Δ^n_ :→ tadpole[-n]
```

Out[25]= $8\left(m^2\right)^{-\epsilon}\left(-2\,\text{two}+\text{one}\,(2-2\epsilon)+(-\text{three}+\text{two})\,(4-2\epsilon)\right)(2-2\epsilon)\,\text{Gamma}[\epsilon]+16\left(m^2\right)^{-\epsilon}\left(-2\,\text{three}+\text{one}\,(1-2\epsilon)+\text{two}\,(1-2\epsilon)\right)\text{Gamma}[1+\epsilon]+\frac{32}{3}\left(m^2\right)^{-\epsilon}(\text{one}+\text{three}+\text{two})\,\text{Gamma}[2+\epsilon]$

This has to vanish! There is no correponding operator in the Euler-Heisenberg Lagrangian!

In[26]:=
```
orderZero2 // FullSimplify
```

Out[26]= $\frac{16}{3}\left(m^2\right)^{-\epsilon}(\text{one}-2\,\text{three}+\text{two})\,(-2+\epsilon)\,(-3+2\epsilon)\,\text{Gamma}[\epsilon]$

OK!

In[27]:=
```
orderOne = Collect[Coefficient[structure1c, λ, 1],
   {λ, Δ}, Simplify]
```

Out[27]= $\frac{1}{\Delta^5}128\,m^4\left((\text{one}+\text{three}+2\,\text{two})\,k\cdot p1+(-\text{one}+\text{three}-2\,\text{two})\,k\cdot p2\right)+\frac{1}{\Delta^3}8\,(-2+d)\left(((-2+d)\,\text{one}-d\,\text{three}-4\,\text{two}+2\,d\,\text{two})\,k\cdot p1-((-2+d)\,\text{one}+d\,\text{three}-4\,\text{two}+2\,d\,\text{two})\,k\cdot p2\right)+\frac{1}{\Delta^4}32\,m^2\left((5\,\text{one}-2\,d\,\text{one}+3\,\text{three}+9\,\text{two}-3\,d\,\text{two})\,k\cdot p1+((-5+2\,d)\,\text{one}+3\,(\text{three}+(-3+d)\,\text{two}))\,k\cdot p2\right)$

Note that, by construction, all order one terms are odd in k_μ and vanish!

In[28]:=
```
orderTwo = Collect[Coefficient[structure1c, λ, 2],
  {λ, Δ}, Simplify]
```

Out[28]=
$$\frac{1}{\triangle^6} 256\, m^4 \left((\text{one} + \text{three} + 3\,\text{two})\,(k\cdot p1)^2 + (-\text{one} + \text{three} - 5\,\text{two})\, k\cdot p1\, k\cdot p2 + (\text{one} + \text{three} + 3\,\text{two})\,(k\cdot p2)^2\right) + \frac{1}{\triangle^3} 8\,(-2+d)\,((-2+d)\,\text{one} + (-8+d)\,\text{three} + (-2+d)\,\text{two})\, p1\cdot p2 + \frac{1}{\triangle^5} 64\, m^2 \left(((5-2\,d)\,\text{one} + 3\,\text{three} - 4\,(-3+d)\,\text{two})\,(k\cdot p1)^2 + 2\,((-2+d)\,\text{one} + \text{three} + 3\,(-3+d)\,\text{two})\, k\cdot p1\, k\cdot p2 + ((5-2\,d)\,\text{one} + 3\,\text{three} - 4\,(-3+d)\,\text{two})\,(k\cdot p2)^2 + 2\, m^2\, \text{two}\, p1\cdot p2\right) + \frac{1}{\triangle^4} 8 \left(2\,(-2+d)\,((-2+d)\,\text{one} - d\,\text{three} - 4\,\text{two} + 2\,d\,\text{two})\,(k\cdot p1)^2 - 2\,(-2+d)\,((-2+d)\,\text{one} + (-4+d)\,\text{three} + 3\,(-2+d)\,\text{two})\, k\cdot p1\, k\cdot p2 + 2\,(-2+d)\,((-2+d)\,\text{one} - d\,\text{three} - 4\,\text{two} + 2\,d\,\text{two})\,(k\cdot p2)^2 - 4\, m^2\,((-1+d)\,\text{one} + (-3+d)\,\text{three} + (-5+2\,d)\,\text{two})\, p1\cdot p2\right)$$

In[66]:=
```
orderTwo2 = Explicit[orderTwo // Expand, k];
```

In[30]:=
$$\text{average} = \left\{k_{\mu 1\_}\, k_{\mu 2\_} :\!\!\to \frac{1}{d}\, g_{\mu 1, \mu 2}\, k\cdot k\right\};$$

In[31]:=
```
orderTwo3 =
  Collect[
   (orderTwo2 /. average // γSimp) /. k · k → Δ + m^2 /.
    onShell, Δ, Simplify]
```

Out[31]=
$$-\frac{256\, m^6\, (\text{one} - \text{three} + 5\,\text{two})\; p1 \cdot p2}{d\,\Delta^6} + \frac{128\, m^4\, ((-4+d)\,\text{one} + 3\,\text{three} + (-19+4\,d)\,\text{two})\; p1 \cdot p2}{d\,\Delta^5} - \frac{1}{d\,\Delta^4}16\,m^2\left(\left(20 - 14\,d + 3\,d^2\right)\text{one} + 84\,\text{two} + d^2\,(3\,\text{three} + 7\,\text{two}) - 2\,d\,(6\,\text{three} + 23\,\text{two})\right) p1 \cdot p2 + \frac{1}{d\,\Delta^3}8\,(-2+d)\left((-2+d)^2\,\text{one} + \left(8 - 10\,d + d^2\right)\text{three} + \left(12 - 8\,d + d^2\right)\text{two}\right) p1 \cdot p2$$

In[32]:=
```
orderTwo4 =
  orderTwo3 /. d → 4 - 2 ϵ /. Δ^n_ :→ tadpole[-n] //
   FullSimplify
```

Out[32]=
$$-\frac{8}{15}\left(m^2\right)^{-1-\epsilon}(-3+2\,\epsilon)\,(4\,\text{one}\,(-3+\epsilon) - 5\,\text{two}\,(1+\epsilon) + \text{three}\,(17+\epsilon))\; p1 \cdot p2\;\text{Gamma}[1+\epsilon]$$

In[33]:=
```
orderTwo4 /. {one → 1, two → 1, three → 1} // Simplify
```

Out[33]= 0

Yes, this works, too!

Third order vanishes; odd in k_μ

In[34]:=
```
orderFour =
  Collect[Coefficient[
    structure1c /. {one → 1, two → 1, three → 1}, λ, 4],
   {λ, Δ}, Simplify];
```

In[67]:= orderFour2 = Explicit[orderFour // Expand, k];

In[36]:= average4 = $k_{\mu 1\_}\, k_{\mu 2\_}\, k_{\mu 3\_}\, k_{\mu 4\_} \to \frac{(k\cdot k)^2}{d\,(2+d)}\,(g_{\mu 1,\mu 2}\, g_{\mu 3,\mu 4} + g_{\mu 1,\mu 3}\, g_{\mu 2,\mu 4} + g_{\mu 1,\mu 4}\, g_{\mu 2,\mu 3})$;

In[37]:= orderFour3 =
Collect[
((orderFour2 // Expand) /. average4 /. average //
γSimp) /. $k\cdot k \to \Delta + m^2$ /. onShell, Δ, Simplify]

Out[37]=
$$\frac{43\,008\, m^8\,(p1\cdot p2)^2}{d\,(2+d)\,\Delta^8} - \frac{512\,(-212+33\,d)\, m^6\,(p1\cdot p2)^2}{d\,(2+d)\,\Delta^7} + \frac{128\,(718-251\,d+25\,d^2)\, m^4\,(p1\cdot p2)^2}{d\,(2+d)\,\Delta^6} - \frac{64\,(-448+276\,d-64\,d^2+5\,d^3)\, m^2\,(p1\cdot p2)^2}{d\,(2+d)\,\Delta^5} + \frac{16\,(144-152\,d+64\,d^2-14\,d^3+d^4)\,(p1\cdot p2)^2}{d\,(2+d)\,\Delta^4}$$

In[38]:= orderFour4 =
orderFour3 /. $d \to 4 - 2\,\epsilon$ /. $\Delta^{n\_} \mapsto$ tadpole[-n] //
FullSimplify

Out[38]=
$$\frac{8\,(m^2)^{-\epsilon}\,(-19+\epsilon\,(7+6\,\epsilon))\,(p1\cdot p2)^2\,\mathrm{Gamma}[2+\epsilon]}{45\, m^4}$$

In[39]:= resultStructure1 =
Series[orderFour4 /. {one → 1, two → 1, three → 1},
{ϵ, 0, 0}] // Normal

Out[39]=
$$-\frac{152\,(p1\cdot p2)^2}{45\, m^4}$$

Now look at the terms order by order: structure 2

EFT calculation

Feynman rules for Euler Heisenberg

From notebook **feynman_rule_Euler_Heisenberg.nb** : feynman rule for Euler Heisenberg Lagrangian. Here the k_i's are the photon momenta.

In[57]:=
```
feynC1 = 32 k1μ4 k2μ3 k3μ2 k4μ1 + 32 k1μ3 k2μ4 k3μ1 k4μ2 +
    32 k1μ2 k2μ1 k3μ4 k4μ3 - 32 k1 · k2 k3μ4 k4μ3 gμ1,μ2 -
    32 k1 · k3 k2μ4 k4μ2 gμ1,μ3 - 32 k1 · k4 k2μ3 k3μ2 gμ1,μ4 -
    32 k2 · k3 k1μ4 k4μ1 gμ2,μ3 + 32 k1 · k4 k2 · k3 gμ1,μ4 gμ2,μ3 -
    32 k2 · k4 k1μ3 k3μ1 gμ2,μ4 + 32 k1 · k3 k2 · k4 gμ1,μ3 gμ2,μ4 -
    32 k3 · k4 k1μ2 k2μ1 gμ3,μ4 + 32 k1 · k2 k3 · k4 gμ1,μ2 gμ3,μ4;
```

$$\text{feynC1} = 32\, k1_{\mu 4}\, k2_{\mu 3}\, k3_{\mu 2}\, k4_{\mu 1} + 32\, k1_{\mu 3}\, k2_{\mu 4}\, k3_{\mu 1}\, k4_{\mu 2} + 32\, k1_{\mu 2}\, k2_{\mu 1}\, k3_{\mu 4}\, k4_{\mu 3} - 32\, k1\cdot k2\, k3_{\mu 4}\, k4_{\mu 3}\, g_{\mu 1,\mu 2} - 32\, k1\cdot k3\, k2_{\mu 4}\, k4_{\mu 2}\, g_{\mu 1,\mu 3} - 32\, k1\cdot k4\, k2_{\mu 3}\, k3_{\mu 2}\, g_{\mu 1,\mu 4} - 32\, k2\cdot k3\, k1_{\mu 4}\, k4_{\mu 1}\, g_{\mu 2,\mu 3} + 32\, k1\cdot k4\, k2\cdot k3\, g_{\mu 1,\mu 4}\, g_{\mu 2,\mu 3} - 32\, k2\cdot k4\, k1_{\mu 3}\, k3_{\mu 1}\, g_{\mu 2,\mu 4} + 32\, k1\cdot k3\, k2\cdot k4\, g_{\mu 1,\mu 3}\, g_{\mu 2,\mu 4} - 32\, k3\cdot k4\, k1_{\mu 2}\, k2_{\mu 1}\, g_{\mu 3,\mu 4} + 32\, k1\cdot k2\, k3\cdot k4\, g_{\mu 1,\mu 2}\, g_{\mu 3,\mu 4};$$

In[58]:=

$$
\begin{aligned}
\text{feynC2} = {} & 8\, k1_{\mu 3}\, k2_{\mu 4}\, k3_{\mu 2}\, k4_{\mu 1} + 8\, k1_{\mu 2}\, k2_{\mu 3}\, k3_{\mu 4}\, k4_{\mu 1} + \\
& 8\, k1_{\mu 4}\, k2_{\mu 3}\, k3_{\mu 1}\, k4_{\mu 2} + 8\, k1_{\mu 3}\, k2_{\mu 1}\, k3_{\mu 4}\, k4_{\mu 2} + \\
& 8\, k1_{\mu 2}\, k2_{\mu 4}\, k3_{\mu 1}\, k4_{\mu 3} + 8\, k1_{\mu 4}\, k2_{\mu 1}\, k3_{\mu 2}\, k4_{\mu 3} + \\
& 8\, k3 \cdot k4\, k1_{\mu 4}\, k2_{\mu 3}\, g_{\mu 1,\mu 2} + 8\, k3 \cdot k4\, k1_{\mu 3}\, k2_{\mu 4}\, g_{\mu 1,\mu 2} - \\
& 8\, k2 \cdot k4\, k1_{\mu 3}\, k3_{\mu 4}\, g_{\mu 1,\mu 2} - 8\, k1 \cdot k4\, k2_{\mu 3}\, k3_{\mu 4}\, g_{\mu 1,\mu 2} - \\
& 8\, k2 \cdot k3\, k1_{\mu 4}\, k4_{\mu 3}\, g_{\mu 1,\mu 2} - 8\, k1 \cdot k3\, k2_{\mu 4}\, k4_{\mu 3}\, g_{\mu 1,\mu 2} - \\
& 8\, k3 \cdot k4\, k1_{\mu 2}\, k2_{\mu 4}\, g_{\mu 1,\mu 3} + 8\, k2 \cdot k4\, k1_{\mu 4}\, k3_{\mu 2}\, g_{\mu 1,\mu 3} - \\
& 8\, k1 \cdot k4\, k2_{\mu 4}\, k3_{\mu 2}\, g_{\mu 1,\mu 3} + 8\, k2 \cdot k4\, k1_{\mu 2}\, k3_{\mu 4}\, g_{\mu 1,\mu 3} - \\
& 8\, k2 \cdot k3\, k1_{\mu 4}\, k4_{\mu 2}\, g_{\mu 1,\mu 3} - 8\, k1 \cdot k2\, k3_{\mu 4}\, k4_{\mu 2}\, g_{\mu 1,\mu 3} - \\
& 8\, k3 \cdot k4\, k1_{\mu 2}\, k2_{\mu 3}\, g_{\mu 1,\mu 4} - 8\, k2 \cdot k4\, k1_{\mu 3}\, k3_{\mu 2}\, g_{\mu 1,\mu 4} + \\
& 8\, k2 \cdot k3\, k1_{\mu 3}\, k4_{\mu 2}\, g_{\mu 1,\mu 4} - 8\, k1 \cdot k3\, k2_{\mu 3}\, k4_{\mu 2}\, g_{\mu 1,\mu 4} + \\
& 8\, k2 \cdot k3\, k1_{\mu 2}\, k4_{\mu 3}\, g_{\mu 1,\mu 4} - 8\, k1 \cdot k2\, k3_{\mu 2}\, k4_{\mu 3}\, g_{\mu 1,\mu 4} - \\
& 8\, k3 \cdot k4\, k1_{\mu 4}\, k2_{\mu 1}\, g_{\mu 2,\mu 3} - 8\, k2 \cdot k4\, k1_{\mu 4}\, k3_{\mu 1}\, g_{\mu 2,\mu 3} + \\
& 8\, k1 \cdot k4\, k2_{\mu 4}\, k3_{\mu 1}\, g_{\mu 2,\mu 3} + 8\, k1 \cdot k4\, k2_{\mu 1}\, k3_{\mu 4}\, g_{\mu 2,\mu 3} - \\
& 8\, k1 \cdot k3\, k2_{\mu 4}\, k4_{\mu 1}\, g_{\mu 2,\mu 3} - 8\, k1 \cdot k2\, k3_{\mu 4}\, k4_{\mu 1}\, g_{\mu 2,\mu 3} + \\
& 8\, k1 \cdot k3\, k2 \cdot k4\, g_{\mu 1,\mu 4}\, g_{\mu 2,\mu 3} + \\
& 8\, k1 \cdot k2\, k3 \cdot k4\, g_{\mu 1,\mu 4}\, g_{\mu 2,\mu 3} - 8\, k3 \cdot k4\, k1_{\mu 3}\, k2_{\mu 1}\, g_{\mu 2,\mu 4} - \\
& 8\, k1 \cdot k4\, k2_{\mu 3}\, k3_{\mu 1}\, g_{\mu 2,\mu 4} - 8\, k2 \cdot k3\, k1_{\mu 3}\, k4_{\mu 1}\, g_{\mu 2,\mu 4} + \\
& 8\, k1 \cdot k3\, k2_{\mu 3}\, k4_{\mu 1}\, g_{\mu 2,\mu 4} + 8\, k1 \cdot k3\, k2_{\mu 1}\, k4_{\mu 3}\, g_{\mu 2,\mu 4} - \\
& 8\, k1 \cdot k2\, k3_{\mu 1}\, k4_{\mu 3}\, g_{\mu 2,\mu 4} + 8\, k1 \cdot k4\, k2 \cdot k3\, g_{\mu 1,\mu 3}\, g_{\mu 2,\mu 4} + \\
& 8\, k1 \cdot k2\, k3 \cdot k4\, g_{\mu 1,\mu 3}\, g_{\mu 2,\mu 4} - 8\, k2 \cdot k4\, k1_{\mu 2}\, k3_{\mu 1}\, g_{\mu 3,\mu 4} - \\
& 8\, k1 \cdot k4\, k2_{\mu 1}\, k3_{\mu 2}\, g_{\mu 3,\mu 4} - 8\, k2 \cdot k3\, k1_{\mu 2}\, k4_{\mu 1}\, g_{\mu 3,\mu 4} + \\
& 8\, k1 \cdot k2\, k3_{\mu 2}\, k4_{\mu 1}\, g_{\mu 3,\mu 4} - 8\, k1 \cdot k3\, k2_{\mu 1}\, k4_{\mu 2}\, g_{\mu 3,\mu 4} + \\
& 8\, k1 \cdot k2\, k3_{\mu 1}\, k4_{\mu 2}\, g_{\mu 3,\mu 4} + 8\, k1 \cdot k4\, k2 \cdot k3\, g_{\mu 1,\mu 2}\, g_{\mu 3,\mu 4} + \\
& 8\, k1 \cdot k3\, k2 \cdot k4\, g_{\mu 1,\mu 2}\, g_{\mu 3,\mu 4};
\end{aligned}
$$

Tree-level in EFT

In[59]:=

```
kin = {k1 → p1, k2 → p2, k3 → p1, k4 → p2};
```

In[60]:=
```
structure1EFT =
  (projector1 (feynC1 C1 + feynC2 C2) /. kin // γSimp) /.
      p1 · p2 → s / 2 /. onShell /. d → 4 // Simplify
```

Out[60]= $8\ (13\ C1 + 6\ C2)\ s^2$

In[61]:=
```
structure2EFT =
  (projector2 (feynC1 C1 + feynC2 C2) /. kin // γSimp) /.
      p1 · p2 → s / 2 /. onShell /. d → 4 // Simplify
```

Out[61]= $8\ (4\ C1 + 3\ C2)\ s^2$

Final result

Matching

Factor (-1) from closed fermion loop

In[62]:=
```
resFinal =
  1/i (-1) e^4 preFactor {resultStructure1,
        resultStructure2} /. p1 · p2 → s / 2 /.
    e → Sqrt[4 π α] /. d → 4
```

Out[62]= $\left\{\frac{38\ s^2\ \alpha^2}{45\ m^4}, \frac{44\ s^2\ \alpha^2}{45\ m^4}\right\}$

From notebook **feynman_rule_Euler_Heisenberg.nb** : EFT result computed with Euler Heisenberg Lagrangian

In[63]:=
```
EFTresult = 1/m^4 {structure1EFT, structure2EFT}
```

Out[63]= $\left\{\frac{8\ (13\ C1 + 6\ C2)\ s^2}{m^4}, \frac{8\ (4\ C1 + 3\ C2)\ s^2}{m^4}\right\}$

In[64]:=
```
matchingRelations = (0 == #) & /@ (resFinal - EFTresult)
```

Out[64]=

$$\left\{0 == -\frac{8\,(13\,C1 + 6\,C2)\,s^2}{m^4} + \frac{38\,s^2\,\alpha^2}{45\,m^4},\; 0 == -\frac{8\,(4\,C1 + 3\,C2)\,s^2}{m^4} + \frac{44\,s^2\,\alpha^2}{45\,m^4}\right\}$$

In[65]:=
```
couplings = Solve[matchingRelations, {C1, C2}][[1]]
```

Out[65]=

$$\left\{C1 \to -\frac{\alpha^2}{36},\; C2 \to \frac{7\,\alpha^2}{90}\right\}$$