4.3. Effective weak Hamiltonian (Fermi theory)

Let us now discuss the weak interaction at low energies. In the standard model, the weak and electromagnetic interactions are described by an $SU_L(2) \times U_Y(1)$ gauge theory, which is broken down to $U_{e.m.}(1)$ by the Higgs mechanism, which gives masses to the $W^\pm$ and Z^0 bosons. A detailed discussion of the standard Model is beyond the scope of this lecture. The only information needed for our discussion is the coupling of $W^\pm$ bosons to fermions. It has the form

$$\mathcal{L}_{cc} = \frac{g_2}{2\sqrt{2}} \left(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu} \right)$$

(g_2: SU(2) coupling)

where $J^+_\mu = (J^-_\mu)^+ = (\bar{u} d')_{V-A} + (c s')_{V-A} + (t b')_{V-A}$

$$+ (\bar{\nu}_e e)_{V-A} + (\bar{\nu}_\mu \mu)_{V-A} + (\bar{\nu}_\tau \tau)_{V-A}$$

and $(\bar{u} d')_{V-A} = \bar{u} \gamma^\mu (1 - \gamma_5) d'$, etc.

The states d', s', b' are not mass eigenstates, ie. the quadratic part of $\mathcal{L}_{SM}$ is not diagonal. The CKM matrix connects (d', s', b') to the mass eigenstates (d, s, b)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo - Kobayashi - Maskawa matrix.

Nobel-Prize in '08.

The matrix is unitary $V_{CKM} V_{CKM}^{+} = \mathbb{1}$. It can further be simplified by phase redefinitions of the fermion fields in $\mathcal{L}_{CC}$ and has

$$3 \times 3 - (2 \times 3 - 1) = 4$$

physical parametrs. An approximate parameterization is the Wolfenstein parameterization, which was designed to show the hierarchy of the different matrix elements.

$$\hat{V}_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

this form is only approximately unitarity

$\lambda = 0,225$; $\rho \cong 0,139$

$A = 0,81$; $\eta \cong 0,342$

The parameters of the matrix correspond to three rotations and one complex phase, which leads to CP violation.

Similarly, the neutrino's ν_e, ν_μ and ν_τ are not mass eigenstates. The corresponding mixing matrix is called PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata), but will not play a role in what follows.

Let us first work at tree level and neglect QCD effects. Then the effective weak Lagrangian can be obtained by integrating out the $W^\pm$ and Z field.

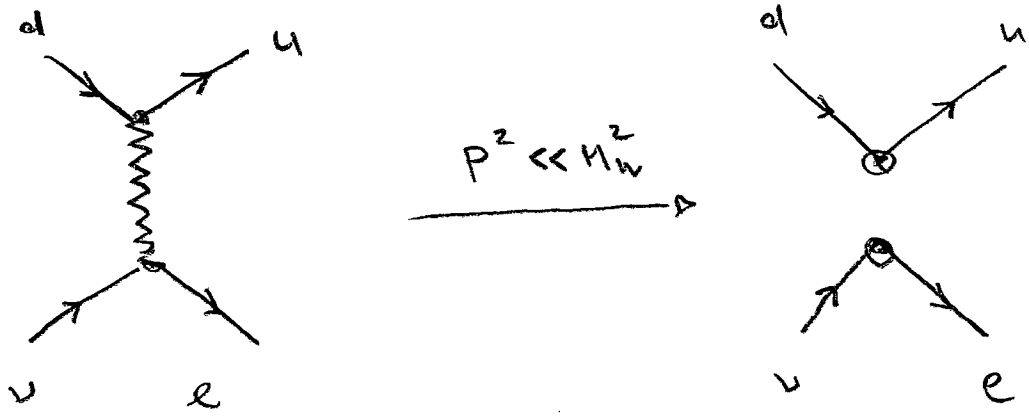
The resulting effective Lagrangian is

$$\mathcal{L}_{eff} = -\frac{g^2}{8M_W^2}\left[J_\mu^- J^{+\mu} + \frac{1}{M_W^2} J_\mu^- (\partial^\mu\partial^\nu - g^{\mu\nu}\Box) J_\nu^+ + \dots \right]$$

where $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$ defines the Fermi constant

$G_F = 1.166 \cdot 10^{-5} (GeV)^{-2}$.

Diagrammatically, this arises from



where the W-propagator gets expanded as

$$\frac{-i}{p^2 - M_W^2}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{M_W^2} \right] = \frac{i}{M_W^2}\left[g^{\mu\nu} - \frac{1}{M_W^2}(p^\mu p^\nu - p^2 g^{\mu\nu}) + O\left(\frac{1}{M_W^4}\right) \right]$$

So already the leading terms in $\mathcal{L}_{eff}$ are irrelevant operators of $d=6$. (The fermion field has $d=\frac{3}{2}$.) Indeed the coefficient of the four-fermion operators $G_F \sim \frac{1}{M_W^2}$ show the expected behavior. The fact that there are no marginal or relevant operators explains the apparent weakness of the interaction at low energies. At higher energies, on the other hand, the weak interaction effects are as strong as electromagnetic interactions. Because of the $\frac{1}{M_W^2}$ suppression, the leading power $d=6$ terms are good enough for most applications. Since it changes lepton and quark flavors, $\mathcal{L}_{eff}$ governs all decays of heavy leptons and hadrons, such as

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

$$n \to p + e^- + \bar{\nu}_e$$

The PDG lists hundreds of pages of various hadron decays. In the SM all of those are governed by G_F and the four parameters in the CKM. If one manages to evaluate the strong interaction effects in such decays, they offer many opportunities to search for new physics effects. An important step is to include QCD corrections to the effective Lagrangian. To do so, we'll have to

i.) Include a complete set of operators, not only those present at tree level.

ii.) perform a matching computation to obtain the Wilson coefficients

iii.) Solve the RG equation for the coefficients to resum large logs.

We'll now discuss two simple examples and then sketch how the construction works in the general case.

1. Example : leptonic decays

Let's consider the operator relevant for $\pi^- \to \mu^- \bar{\nu}$, which is based on the quark level transition $\bar{u}d \to \bar{\nu}_\mu \mu^-$. The tree-level Left is

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} (\bar{u}d)_{V-A} (\bar{\mu}\nu_\mu)_{V-A}$$

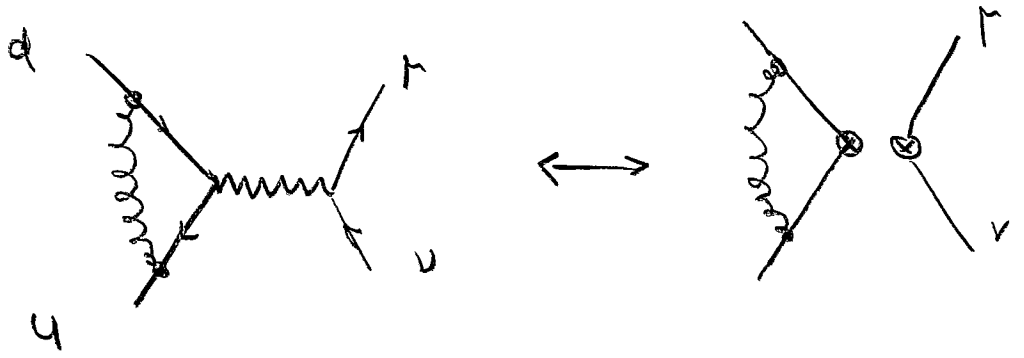
It turns out that this is the only operator with this flavor structure: the weak interactions only couple to left-handed fields $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$, and the QCD interactions conserve helicity for vanishing quark masses $\mathcal{L}_{QCD}^{(massless)} = \bar{\psi}_L i\not{D} \psi_L + \bar{\psi}_R i\not{D}_R \psi_R$.

Chirality violating operators are suppressed by powers of the quark masses. The only possible Dirac bilinear is $\bar{\psi}_L \gamma^\mu \psi_L = -\bar{\psi}_L \gamma^\mu \gamma^5 \psi_L$ and $\bar{\psi}_L \psi_L = \bar{\psi}_L \sigma^{\mu\nu} \psi_L = 0$. This leaves as operators only

$$\bar{u}_L \gamma^\mu d_L \; \bar{\mu}_L \gamma_\mu \nu_L = \bar{u}_L \gamma^\mu \nu_L \; \bar{\mu}_L \gamma_\mu d_L$$

that the two operators are equal is an example of a Fierz identity.

Not only is there only a single operator, but also the matching is trivial since all QCD corrections are the same in the full and effective theory



So there are no QCD corrections to $\mathcal{L}_{\text{eff}}$. QCD effects only arise in the matrix element

$$\langle 0 | \bar{u} \gamma^\mu (1-\gamma_5) d | \pi^- \rangle \qquad \text{(130 MeV)}$$

$$= - \langle 0 | \bar{u} \gamma^\mu \gamma^5 d | \pi^-(p) \rangle = - i f_\pi p^\mu$$

$$= -i \sqrt{2} F_\pi p^\mu$$

(92 MeV)

The matrix element f_π can be obtained from lattice calculations.

By measuring the π-decay rate

$$\Gamma(\pi \to l\nu) = \frac{G_F}{8\pi^2} f_\pi^2 m_l^2 m_\pi \left(1 - \frac{m_l^2}{m_\pi^2}\right) |V_{ud}|^2$$

one can then determine $|V_{ud}|$.

Similarly, one obtains $|V_{cd}|$ from $D^- \to \mu^- \bar{\nu}$ and $|V_{ub}|$ from $B^- \to \tau^- \bar{\nu}$, and $|V_{us}|$ from $K^- \to \mu^- \bar{\nu}$.

2. Example: hadronic decays

Let's consider the decay $\bar{B}^{(0)} \to D_s^- \pi^+$, based on the $b \to u \bar{c} s$ quark transition. In this case there are two operators which differ by their color structure

$$O_1 = \bar{s}_L^i \gamma_\mu c_L^i \; \bar{u}_L^j \gamma^\mu b_L^j$$

$$O_2 = \bar{s}_L^i \gamma_\mu c_L^j \; \bar{u}^j \gamma^\mu b_L^i$$

Where the color indices i and j are summed over. Note that

$$\bar{s}_L \gamma_\mu t^a c \; \bar{u} \gamma^\mu t^a b = \frac{1}{2} O_2 - \frac{1}{2N} O_1 .$$

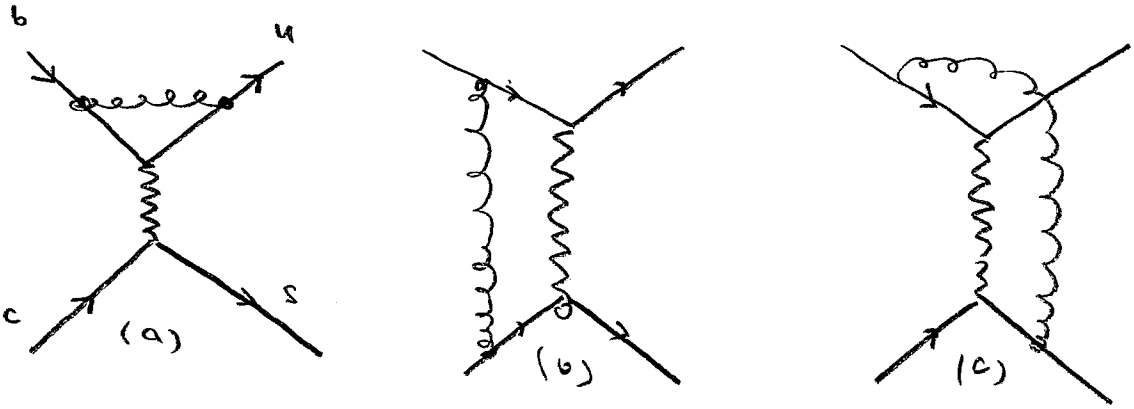
As we will see, the matching is now non trivial and we write

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu) O_1 + C_2(\mu) O_2 \right]$$

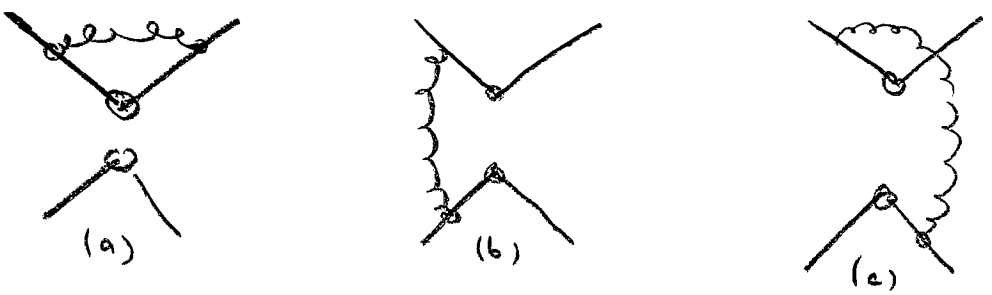
At tree level, one has $C_1 = 1$, $C_2 = 0$.

To obtain the one-loop coefficients one performs a matching computation, i.e. one computes the four quark Green's function in both the full and the effective theory:

Full theory:



EFT



+ "mirrored" diagrams

The difference between the full and effective theory results is absorbed into C_1 and C_2.

Since C_1, C_2 are independent of m_q, we can set all quark masses to zero. Furthermore, also any values for the external momenta will work. The simplest choice is $p_i = 0$. In this case (a) in the full and all diagrams in the effective theory vanish, because they are scaleless:

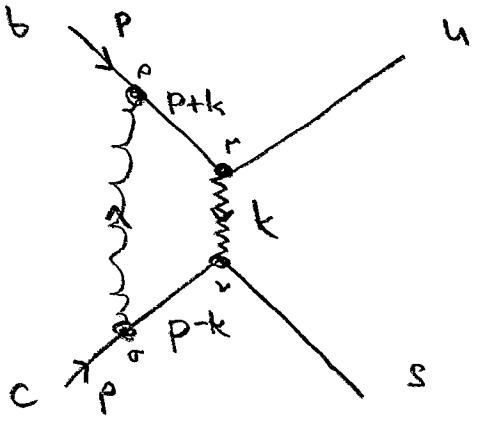
$$\int d^4k \, \frac{1}{k^2} \frac{1}{\not{k}} p \frac{1}{\not{k}} r = 0 = \frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}$$

the infrared divergences cancel in the matching.

We will for pedagogical reasons keep p_i nonzero but use the same value for all legs.

Let's consider (6) in the full theory

(b) =



$$= \frac{g_2^2/8}{\sqrt{2}} V_{cs}^* V_{ub} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2} \frac{-i}{k^2 - M_W^2} \frac{i}{(p+k)^2} \frac{i}{(p-k)^2}$$

$$\cdot \bar{u} \cdot \gamma^\mu (1-\gamma_5)(\not{p}+\not{k})\, i g t^a \gamma^\rho\, b$$

$$\cdot \bar{s} \cdot \gamma^\nu (1-\gamma_5)(\not{p}-\not{k})\, i g t^a \gamma^\sigma\, c$$

$$\cdot g^{\rho\sigma} \left[g^{\mu\nu} - \frac{k^\mu k^\nu}{M_W^2} \right]$$

Some remarks: 1.) This is an amputated Green's function, the spinors $\bar{u}\,\Gamma_1\, b\, \bar{s}\,\Gamma_2\, c$ are only there to remind ourselves which color and spin index goes where.

2.) I'm using Feynman gauge for QCD an unitary gauge for the W-propagator. The $k^\mu k^\nu \frac{1}{M_W^2}$ - term

does not contribute to the sum of the diagrams so let's omit it.

3.) The color structure is $t^a \otimes t^a$. To rewrite this in the form of O_1 and O_2, use

$$(t^a)_{ij}\,(t^a)_{ke} = \frac{1}{2}\,\delta_{ie}\,\delta_{kj} - \frac{1}{2N_c}\,\delta_{ij}\,\delta_{ke}$$

4.) To simplify the Dirac structure one needs identities such as $\left[\Gamma = \gamma^\rho(1-\gamma^5)\right]$

$$\Gamma\,\gamma_\rho\gamma_\tau \otimes \Gamma\,\gamma^\rho\gamma^\tau = 16\,\Gamma\otimes\Gamma$$

The coefficient can be derived by taking traces

$$\mathrm{tr}\left[A\,\Gamma\,\gamma_\rho\gamma_\tau\,B\,\Gamma\,\gamma^\rho\gamma^\tau\right].$$

$$\mathrm{tr}\left[A\,\Gamma\,B\,\Gamma\right]$$

for some Dirac matrices A and B, e.g. $A = B = \gamma^\mu$.

5.) Without the $k^\mu k^\nu / m_W^2$ - term, the diagram is finite, so we only need it's value for $\alpha = 4$

The diagram (b) in the effective theory, on the other hand is divergent, as it behaves like

$$\int d^dk \; \frac{1}{k^2} \; \frac{1}{\not p - \not k} \; \frac{1}{\not p + \not k} \sim \int \frac{d^dk}{k^4} \sim \frac{1}{\varepsilon} + \ln\left(\frac{\Lambda^2}{p^2}\right)$$

This leads to some technical issues:

1.) The Dirac basis consists all totally antisymmetric products of γ^μ's: γ^μ, $\gamma^{[\mu}\gamma^{\nu]} = \frac{1}{2}[\gamma^\mu, \gamma^\nu] = -i\sigma^{\mu\nu}$

$$\gamma^{[\mu}\gamma^\nu\gamma^\rho] \doteq +i\varepsilon^{\mu\nu\rho\sigma}\gamma^\sigma\gamma^5 \quad \text{(for } d=4\text{)}$$

$$\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^\sigma] = i\gamma^5\varepsilon^{\mu\nu\rho\sigma} \qquad [\varepsilon_{0123} = 1 = -\varepsilon^{0123}$$

In d-dimensions, one can also write down antisymmetric products of more than four γ^μ's, which vanish in four dimensions. Operators with such Dirac structures are called <u>evanescent</u>. One can use a renormalization scheme where their physical matrix elements vanish, but they need to be included in $\mathcal{L}_{\text{eff}}$ for consistency.

2.) γ^5 is special to $d=4$. The rule

$\{\gamma^\mu, \gamma^5\} = 0$ leads to inconsistencies

in d-dimensions.

A prescription to eliminate γ^5 is to replace

it by ('t Hooft & Veltman '72)

$$\gamma_5 = i \frac{1}{4!} \varepsilon_{\nu_1\nu_2\nu_3\nu_4} \gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4}$$

and to perform all computations with

$$\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4}$$

and only contract with the ε-tensor after

renormalization.

Alternatively one can use

- Naive Dimensional Regularization (NDR), i.e. use $0 = \{\gamma^\mu, \gamma^5\}$ and hope that no inconsistency arise.
- or Dimensional Reduction (DRED), i.e. treat γ^μ's and the gauge fields as four dimensional.

Let us now discuss the results. The bare Green's function in the SM and in the EFT are ($d=4-2\varepsilon$)

$$\Gamma_{full} = \frac{G_F V_{cs}^* V_{ud}}{\sqrt{2}} \Big\{ \Big[1 + 2C_F \frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big] \langle O_1\rangle_{tree} + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2}\langle O_1\rangle_{tree} - 3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2}\langle O_2\rangle_{tree} \Big\} + O(p^2/M_W^2)$$

$$\Gamma_{eff} = \frac{G_F V_{cs}^* V_{ud}}{\sqrt{2}} \Big\{ C_1^{bare} \Big[\Big(1 + 2C_F\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big)\langle O_1\rangle_{tree} + \frac{3}{N}\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\langle O_1\rangle_{tree} - 3\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\langle O_2\rangle_{tree} \Big] + C_2^{bare}\Big[\Big(1 + 2C_F\frac{\alpha_s}{4\pi}\Big|\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big)\langle O_2\rangle_{tree} + \frac{3}{N}\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\langle O_2\rangle_{tree} - 3\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\langle O_1\rangle_{tree} \Big] \Big\}$$

The first term corresponds to a diagram (a); the remainder to (b)+(c) (and their mirrored counterparts). The color factors are $N \equiv N_c = 3$ and $C_F = \frac{N^2-1}{2N} = \frac{4}{3}$. $\alpha_s = \frac{g^2}{4\pi}$. The Wilson coefficients have an expansion $C_i = C_i^{(0)} + \frac{\alpha}{4\pi} C_i^{(1)} + \dots$.

The bare amputated Green's functions have divergences. In the full theory, these are removed by wave-function renormalization

$\psi^{(0)} = Z_q^{1/2}\,\psi$. $Z_q = 1 - \frac{\alpha_s}{4\pi} C_F \frac{1}{\varepsilon}$. [This is the $\overline{\text{MS}}$ wave function ren. in Feynman gauge.]

In the effective theory, there are additional divergences from (b) and (c), which are not removed by wave-function renormalization, but by renormalization of the Wilson coefficients in $\mathcal{L}_{\text{eff}}$.

Omitting the overall factor $-\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}$, one has

$$\mathcal{L}_{\text{eff}} = C_i^{\text{bare}} O_i(q^{(0)}) = Z_q^2 \overbrace{C_i Z_{ij}}^{C_j^{\text{bare}}} O_j(q)$$

The renormalization constants form a matrix.

Expanding $Z_{ij} = \delta_{ij} + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon} Z_{ij}^{(1)}$, one finds that

$$Z = \mathbb{1} + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} -3/N & +3 \\ +3 & -3/N \end{pmatrix}$$

removes the remaining divergences in T_{eff}.

From the condition $T_{full} - T_{eff} = 0$, one then reads off C_1 and C_2:

$$C_1(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}$$

$$C_2(\mu) = 0 - 3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}$$

Again, note that C_1 and C_2 only depend on M_W and μ, but not on the low energy scale p^2. This has to be the case: the low-energy physics must drop out in the matching. The last step in the construction of $\mathcal{L}_{eff}$ is to solve the RG equations for C_1 and C_2 to avoid having large log's when evaluating $C_1(\mu)$ at low values of μ.

The RG equation follows from the fact that physical quantities are μ-independent. Equally well, we can use the fact that bare quantities are μ-independent:

$$\mu \frac{d}{d\mu} C_j^{(\text{bare})}(\xi, \alpha^{\text{bare}}) = 0 \,.$$

$$= \mu \frac{d}{d\mu} C_i(\mu) Z_{ij}(\alpha_s(\mu), \xi)$$

$$= \left[\mu \frac{d}{d\mu} C_i(\mu)\right] Z_{ij} + C_i \left[\mu \frac{d}{d\mu} Z_{ij}\right]$$

$$\rightarrow \quad \mu \frac{d}{d\mu} C_j(\mu) - C_i \gamma_{ij}(\alpha) = 0$$

$$\text{with} \quad \gamma_{ij} = \left(\mu \frac{d}{d\mu} Z_{ik}\right) Z^{-1}_{kj}$$

In vector notation

$$\left(\mu \frac{d}{d\mu} - \hat{\gamma}^T\right) \vec{C}(\mu) = 0$$

In the $\overline{MS}$ scheme, the Z-matrix is a sum of pole terms $\hat{Z} = \mathbb{1} + \sum_{k=1}^{\infty} \frac{1}{\varepsilon^k} \hat{Z}_k(\alpha_s)$ and there is a simple relation

$$\hat{\gamma} = 2\alpha_s \frac{\partial \hat{Z}_1}{\partial \alpha_s} = \frac{\alpha_s}{4\pi} \begin{pmatrix} -6/N & +6 \\ +6 & -6/N \end{pmatrix}$$

To solve the RG equation, it is simplest to use a basis, in which $\hat{\gamma}$ is diagonal, in our case $C_\pm = C_1 \pm C_2$ are the corresponding continations:

$$\mu \frac{d}{d\mu} C_\pm(\mu) = \frac{\alpha}{4\pi} 6 \left(\pm 1 - \frac{1}{N} \right) C_\pm(\mu)$$

$$= \frac{\alpha}{4\pi} (-\gamma_{0\pm}) C_\pm(\mu)$$

$$\mu \frac{d\alpha_s}{d\mu} = -2\alpha_s \left[\frac{\alpha_s}{4\pi} \beta_0 + \left(\frac{\alpha}{4\pi}\right)^2 \beta_1 + \cdots \right]$$

$$\Rightarrow \quad \frac{dC}{C} = -d\ln\mu \frac{\alpha_s}{4\pi} \gamma_\pm = -\frac{d\alpha}{\beta(\alpha)} \frac{\alpha_s}{4\pi} \gamma_\pm$$

$$= \frac{d\alpha_s}{-2\alpha\beta_0 \frac{\alpha_s}{4\pi}} \left(-\frac{\alpha_s}{4\pi} \gamma_\pm\right) = + \frac{d\alpha_s}{\alpha_s} \frac{\gamma_\pm}{2\beta_0}$$

$$\Rightarrow \ln\left(\frac{C(\mu)}{C(M_W)}\right) = \ln\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right) \frac{\gamma_\pm}{2\beta_0}$$

or $$C_\pm(\mu) = C_\pm(M_W)\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{+\frac{\gamma_\pm}{2\beta_0}}$$

Then one uses $C_\pm(M_W) = 1 + O(\alpha_s^2)$

and transforms back to the original basis

$$C_1(\mu) = \frac{1}{2}\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{\frac{\gamma_+}{2\beta_0}} + \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-\frac{\gamma_-}{2\beta_0}}\right]$$

$$C_2(\mu) = \frac{1}{2}\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{\frac{\gamma_+}{2\beta_0}} - \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{\frac{\gamma_-}{2\beta_0}}\right]$$

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f \;;\; \gamma_\pm = -6\left(\pm 1 - \frac{1}{N_c}\right)$$

$$= \mp 6 + 2 = \begin{cases} -4 \\ +8 \end{cases}$$

Numerically for $\mu = m_b$:

$$C_1(\mu) = 1,10 \quad , \quad C_2(\mu) = -0,24 .$$

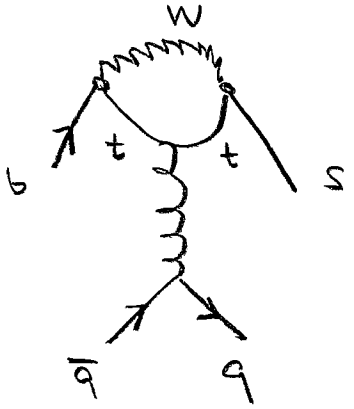
This completes the computation of the QCD effects to $\mathcal{L}_{\text{eff}}$ relevant for $b \to u\bar{c}s$ which mediates $\bar{B} \to D_s^- \pi^+$.

The structure of $\mathcal{L}_{\text{eff}}$ becomes more complicated for FCNC (flavor-changing neutral current) processes, such as

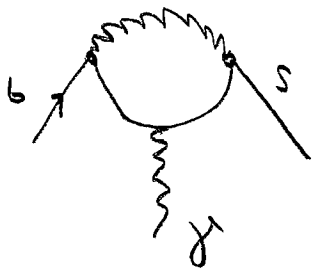
$$b \to s\gamma \quad , \quad b \to sg ,$$

$$b \to s\bar{q}q \quad , \quad b \to s\ell^+\ell^-$$

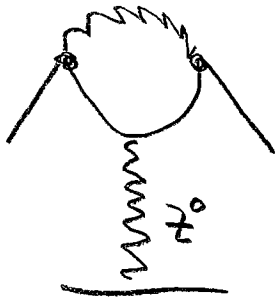
For such decays so called penguin diagrams contribute, e.g.



QCD penguin



electromagnetic penguin



electroweak penguin.

Such processes are interesting, because they can violate CP, and are sensitive to new physics because they only arise at loop-level

in the SM. The effective Lagrangian for such processes contains 12 operators, which all mix under renormalization, i.e.

$$\left[\frac{d}{d\ln\mu} - \gamma^T \right] \vec{C} = 0$$

γ^T: 12 × 12 matrix.

Although there is a lot of interesting phenomenology associated with such decays, we will not discuss them further, but let us note that the corresponding anomalous dimension matrix has been calculated at $O(\alpha_s^3)$, which involved the computation of hundreds of thousands of three and four-loop diagrams.