9.2. Jet production

For the case of DIS and for Drell-Yan, we were able to show that the hadronic cross section is obtained by convoluting the partonic cross section with the parton distribution functions.

Although a formal proof is lacking*, it is generally believed that the same factorization theorem is also valid for more exclusive processes, which involve jets in the final state so that

$$d\sigma(N_1(P_1)+N_2(P_2)\to n \text{ jets}) =$$

$$\int d\xi_1 \int d\xi_2 \sum_{i,j} d\hat{\sigma}(i(\xi_1 P_1)\, j(\xi_2 P_2)\to n \text{ jets})$$

$$\times\, f_{i/N_1}(\xi_1)\, f_{j/N_2}(\xi_2)$$

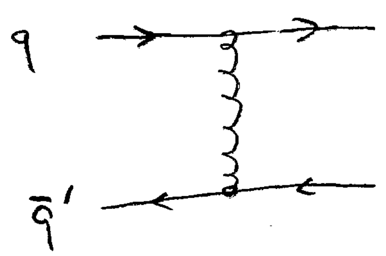
---

* See arXiv: 0808.2191 for a discussion of jet-production in SCET.

As an example, let's consider two-jet production at leading power in α_s. Even in this simple example, a significant number of diagrams contribute.

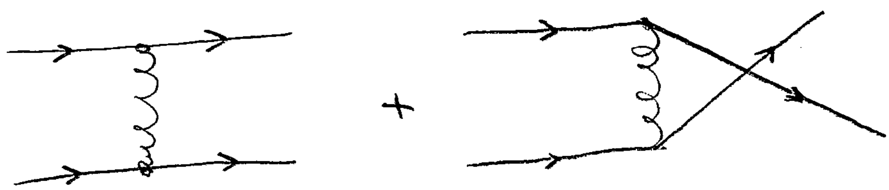
let's draw those channel by channel.

a.) For $i = q$, $j = \bar{q}'$, $q \neq q'$

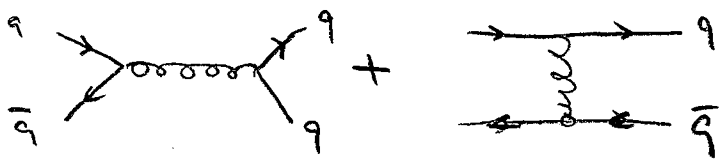


b.) $i = q$, $j = q'$. Same as a.)

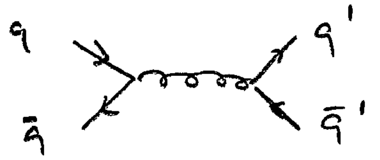
c.) $i = q$, $j = q$



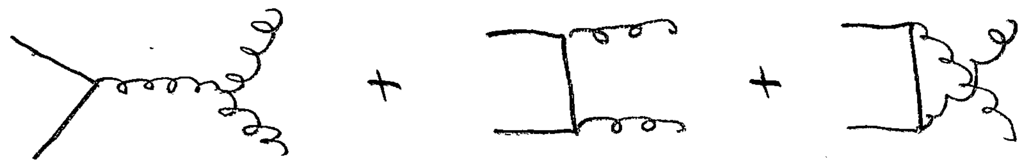
d.) $i = q$, $j = \bar{q}$



for $q\bar{q}$ final state



for $q\bar{q}'$ final state



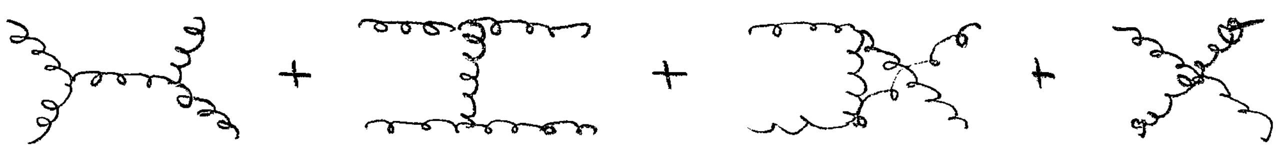
for gg

Note that the diagrams with the same final state interfere, but not the other ones.

$$d\hat{\sigma}_{q\bar{q}} \sim |\mathcal{M}_{q\bar{q}\to q\bar{q}}|^2 + |\mathcal{M}_{q\bar{q}\to q'\bar{q}'}|^2$$

$$+ |\mathcal{M}_{q\bar{q}\to gg}|^2 .$$

e.) $i = g,\ j = g$



for gg final state.

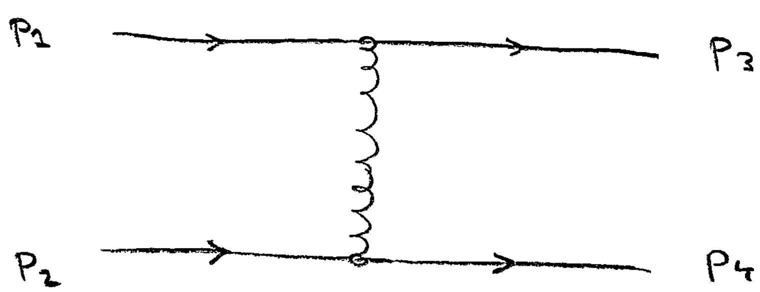
see d.) for the $q\bar{q}$ final state.

f.) $qg \to qg$ and $\bar{q}g \to \bar{q}g$

rotate the $\bar{q}q \to gg$ diagrams

To get the two-jet cross section, we need to apply a jet algorithm to the final state. However, because the final state only contains two particles we'll end up with two jets (unless our cone is so large that we absorb the entire final state into it.)

There is no point in trying to evaluate all these diagrams in the lecture, but let us calculate one of them to illustrate how to deal with the color structure, etc.



Define $\hat{s} = (p_1 + p_2)^2 = (\xi_1 P_1 + \xi_2 P_2)^2 = \xi_1 \xi_2 s$

$\hat{t} = (p_1 - p_3)^2$

$\hat{u} = (p_1 - p_4)^2 \qquad s + t + u = 0.$

(we are neglecting the small quark masses)

$$i\mathcal{M} = (ig)^2 \frac{-i}{(p_1 - p_3)^2} \bar{u}(p_3) \gamma^\mu t^a u(p_1) \bar{u}(p_4) \gamma^\mu t^a u(p_2)$$

To get the cross section, we sum over colors and spins of the out-going particles and average over the incoming.

$$d\sigma \propto \frac{1}{4} \frac{1}{N_c^2} \sum_{\substack{\text{spins}\\ \text{colors}}} |\mathcal{M}|^2$$

$$= \frac{g^4}{4N_c^2} \sum_{\substack{\text{spins}\\ \text{colors}}} \bar{u}(p_3) \gamma^\mu t^a u(p_1) \bar{u}(p_1) \gamma^\nu t^b u(p_3)$$

$$* \; \bar{u}(p_4) \gamma^\mu t^a u(p_2) \bar{u}(p_2) \gamma^\nu t^b u(p_4))$$

$$= \frac{g^4}{4N_c^2} \frac{1}{\hat{t}^2} \operatorname{tr}[\not{p}_3 \gamma^\mu \not{p}_1 \gamma^\nu] \operatorname{tr}[\not{p}_4 \gamma^\mu \not{p}_2 \gamma^\nu]$$

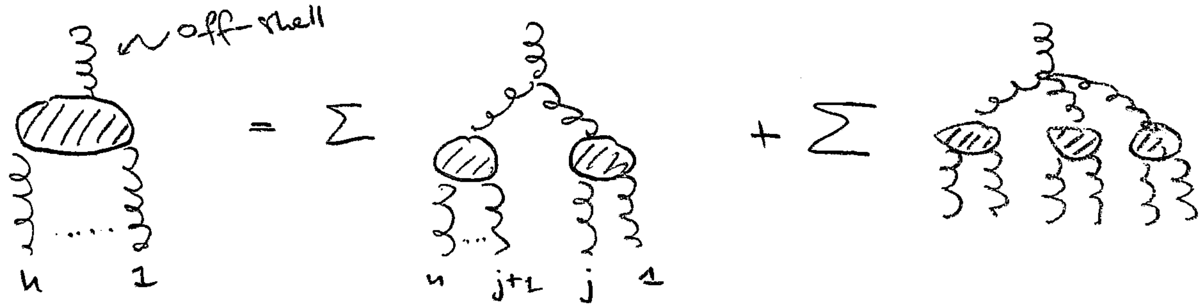
$$\cdot \operatorname{tr}_c[t^a t^b] \operatorname{tr}_c[t^a t^b]$$

$$= \frac{g^4}{4N_c^2} \frac{1}{\hat{t}^2} 8(\hat{s}^2 + \hat{u}^2) \underbrace{\frac{1}{2}\delta^{ab} \frac{1}{2}\delta^{ab}}_{\frac{N^2-1}{4}}.$$

While the computation of these diagrams is straightforward, it quickly becomes quite tedious. Also, for processes with more legs, the number of diagrams quickly increases, e.g. for $gg \to ng$, we have

n	2	3	4	5	6	7	8
diagrams	4	25	220	2485	34300	559'405	10'525'900

For this reason automated codes were developed which generate and calculate the corresponding Feynman diagrams and then integrate them numerically over phase-space. For large n, Feynman diagrams become inefficient and one uses recursion relations, which allow one to obtain amplitudes with more legs by adding a leg to lower-n amplitudes, e.g.



For a review, see 0707.3342 by S. Weinzierl. While these off-shell recursion relations are known since a long time, recently new types of on-shell recursion relations were developed.*

Two general-purpose and user friendly tree-level codes are CompHEP (http://comphep.sinp.msu.ru/) and MadGraph (http://madgraph.hep.uiuc.edu).

see slides for a calculation of $pp \to 2$ jets using MadGraph, MadEvent and MadAnalysis.

---

* Off-shell: Berends & Giele '88

On-shell: Britto, Cachazo, Feng, Witten '04

Cachazo, Svrcek, Witten '04