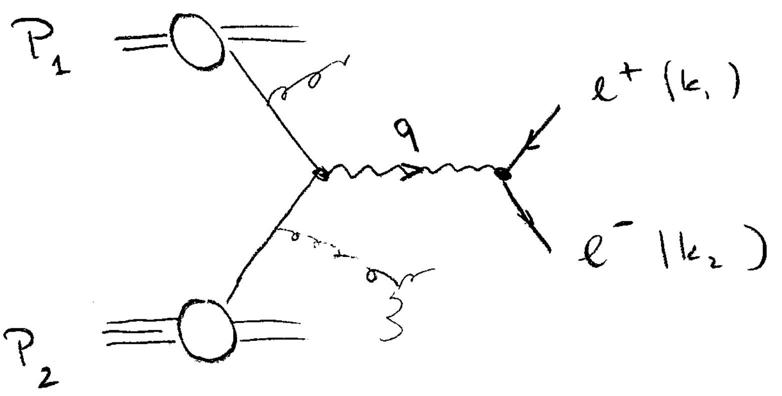
9. Hadron - hadron collisions

9.1. Lepton pair production

One of the most basic processes at a hadron collider is Drell-Yan production, the reaction in which a lepton pair with high invariant mass M emerges after the collision, $pp \to e^+e^- + X$.



$$M^2 = q^2 = (k_1+k_2)^2$$

We work in the C.M.S. and parameterize

$$P_1^\mu = E(1,0,0,1) = E\, n^\mu ,$$

$$P_2^\mu = E(1,0,0,-1) = E\, \bar{n}^\mu .$$

The cross section

$$d\sigma = \frac{1}{2S} \sum_X |\mathcal{M}_{P_1 + P_2 \to k_1 + k_2 + p_X}|^2$$

$$\cdot \frac{d^3 k_1}{2E_1 (2\pi)^3} \frac{d^3 k_2}{2E_2 (2\pi)^3}$$

can be written as a leptonic times a hadronic part.

$$d\sigma = \frac{1}{2S} \frac{e^4}{M^4} \sum_{\text{spins}} \bar{u}(k_1) \gamma^\nu v(k_2) \bar{v}(k_2) \gamma^\mu u(k_2)$$

$$\times \frac{1}{4} \sum_{\text{spins}} \sum_X (2\pi)^4 \delta^{(4)}(P_1 + P_2 - q - q_X)$$

$$\langle P_1 P_2 | J^\mu(0) | X \rangle \langle X | J^\nu(0) | P_1 P_2 \rangle$$

$$= \frac{8\pi^2 \alpha^2}{S M^4} L^{\mu\nu} W_{\mu\nu} .$$

$$L^{\mu\nu} = \sum_{\text{spins}} \bar{u}(k_1) \gamma^\nu v(k_2) \bar{v}(k_2) \gamma^\mu u(k_1)$$

$$= 4 \left[k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - k_1 \cdot k_2 \, g^{\mu\nu} \right]$$

$$W_{\mu\nu} = \frac{1}{4}\sum_{\text{spins}} \int d^4x\, e^{-iqx} \langle P_1 P_2 | J^\mu(x) J^\nu(0) | P_1 P_2 \rangle$$

$$e^{i\hat{P}x} J^\mu(0) e^{-i\hat{P}x}$$

$$= \frac{1}{4}\sum_{\text{spins}} \int d^4x\, e^{-iqx} \sum_X \langle P_1 P_2 | J^\mu(x) | X\rangle\langle X | J^\nu(0) | P_1 P_2 \rangle$$

$$= \frac{1}{4}\sum_{\text{spins}} \int d^4x\, e^{-iqx} \sum_X e^{i(P_1+P_2-P_X)} \langle P_1 P_2 | J^\mu(0) | X\rangle\langle X | J^\nu(0) | P_1 P_2 \rangle$$

$$= \frac{1}{4}\sum_{\text{spins}} (2\pi)^4 \delta^{(4)}(P_1+P_2-P_X) \langle P_1 P_2 | J^\mu(0) | X\rangle\langle X | J^\nu(0) | P_1 P_2 \rangle$$

After integrating over the lepton angles, one obtains

$$\frac{d\sigma}{dM^2} = \frac{2\alpha^2}{3M^2 S} \frac{1}{4}\sum_{\text{spin}} \langle P_1 P_2 | W | P_1 P_2 \rangle$$

where

$$W(M) = -\int \frac{d^4q}{(2\pi)^3}\, \Theta(q^0)\, \delta(q^2 - M^2)$$

$$\times \int d^4x\, e^{-iqx} J^\mu(x) J_\mu(0) .$$

Now we'd like to analyze this process in the effective theory. We now have two directions with large energy flow and therefore two sets of collinear fields.

The building blocks for leading-power operators are

$$\left.\begin{matrix} \chi_1 \\ B_{1\perp}^{\mu} \end{matrix}\right\} \begin{matrix} \text{collinear quark and gluon field} \\ \text{large energy in } n\text{-direction} \\ p \mathrel{\hat=} \bar{n}\cdot p \, \frac{n^\mu}{2} \sim E\, n^\mu \end{matrix}$$

$$\left.\begin{matrix} \chi_2 \\ B_{2\perp}^{\mu} \end{matrix}\right\} \begin{matrix} \text{collinear fields in } \bar{n}^\mu \\ \text{direction} \end{matrix}$$

We would like to show that

$$W(M) = \int ds \int dt \; C_{qq}(s,t,M^2)\, \bar{\chi}_1(s\bar{n}) \frac{\bar{n}\!\!\!/}{2} \chi_1(0)\, \bar{\chi}_2(tn) \frac{n\!\!\!/}{2} \chi_2(0)$$

$$+ \int ds \int dt \; C_{qg}(s,t,M^2)\, \bar{\chi}_1(s\bar{n}) \frac{\bar{n}\!\!\!/}{2} \chi_1(0)\, \mathrm{tr}\left[B_{2\perp}^{\mu}(tn)\, B_{2\mu}^{\perp}(0) \right]$$

$$+ \text{"gluon-1, quark-2"} + \text{"gluon-1, gluon-2"}$$

If the operator has this form, then

1.) the soft fields will decouple like in DIS

2.) the proton matrix elements will factor into products of parton distribution functions.

Let us now argue that all non-vanishing contributions are of the above form. First of all, only operators with two fields from each sector have nonvanishing matrix elements, because we take color-singlet matrix elements

$$\langle P_1 | \chi(0) | P_1 \rangle = 0 \quad , \text{etc.}$$

Also, to be able to form a color-singlet we need two gluon fields or two quark fields from each sector.

Let us look at the operator built from four quark fields. It's general form is

$$\mathcal{O} = \bar{\chi}^i_{1,\alpha} \, \chi^j_{1,\beta} \, \bar{\chi}^k_{2,\gamma} \, \chi^l_{2,\delta} \, C^{ijkl}_{\alpha\beta\gamma\delta}$$

Here $\alpha, \beta, \gamma, \delta$ are Dirac indices, i, j, k, l are color indices.

A basis of Dirac matrices Γ_1^A relevant for $\bar{\chi}_1 \Gamma_1^A \chi_1$ is

$\Gamma_1^A \in \{ \frac{\not{k}}{2}, \frac{\not{k}}{2}\gamma_5, \frac{\not{k}}{2}\gamma_\perp^\perp \}$. Because of the constraint

$\not{n}\chi_1 = 0$, only four matrices are needed. The basis for χ_2 is $\Gamma_1^B \in \{ \frac{\not{k}}{2}, \frac{\not{k}}{2}\gamma_5, \frac{\not{k}}{2}\gamma_\perp^\perp \}$.

A general operator has the form

$$\mathcal{O} = \sum_{A,B} C_{AB} \, \bar{\chi}_1^i \Gamma_1^A \chi_1^j \; \bar{\chi}_2^k \Gamma_2^B \chi_2^l \, .$$

Because the operator W is a scalar, $C_{AB} \propto \delta_{AB}$.

Furthermore, each fermion pair can be in a color singlet or a color octet.

The most general form of the four-fermion operator is

$$\mathcal{O} = C_S^1\, \bar{\chi}_1 \frac{\not{n}}{2} \chi_1\, \bar{\chi}_2 \frac{\not{\bar n}}{2} \chi_2$$

$$+ C_P^1\, \bar{\chi}_1 \frac{\not{n}}{2} \gamma_5 \chi_1\, \bar{\chi}_2 \frac{\not{\bar n}}{2} \gamma_5 \chi_2$$

$$+ C_V^1\, \bar{\chi}_1 \frac{\not{n}}{2} \gamma_\mu^\perp \chi_1\, \bar{\chi}_2 \frac{\not{\bar n}}{2} \gamma_\perp^\mu \chi_2$$

$$+ C_S^8\, \bar{\chi}_1 \frac{\not{n}}{2} t^a \chi_1\, \bar{\chi}_2 \frac{\not{\bar n}}{2} t^a \chi_2$$

$$+ C_P^8 (\dots) + C_V^8 (\dots)$$

After performing the decoupling transformation the color-singlet operators factorize

$$\mathcal{O} \to C_S^1\, \bar{\chi}_1^{(0)} \frac{\not{n}}{2} \chi_1^{(0)}\, \bar{\chi}_2^{(0)} \frac{\not{\bar n}}{2} \chi_2^{(0)} + C_P^1 (\dots) + C_V^1$$

$$+ C_S^8\, \bar{\chi}_1^{(0)} S_n^\dagger(0) \frac{\not{n}}{2} t^a S_n(0)\, \chi_1^{(0)}\, \bar{\chi}_2^{(0)} S_{\bar n}^\dagger(0)\, t^a S_{\bar n}(0) \frac{\not{\bar n}}{2} \chi_2^{(0)}$$

$$+ \dots$$

but in the color-octet operators, the soft Wilson lines do <u>not</u> cancel.

We can write

$$S_n^+ t^a S_n = 2\,\mathrm{tr}\left[t^b S_n^+ t^a S_n \right] t^b$$

since $\{ \mathbb{1}, t^b \}$ is a basis of 3x3 matrices. So

$$① = C_S^1 (\ldots) + C_P^1 (\ldots) + C_V^1$$

$$+ C_S^8 \cdot S_{cd}\, \bar{\chi}_1 t^c \frac{\not{k}}{2} \chi_1\, \bar{\chi}_2 t^d \frac{\not{k}}{2} \chi_2$$

$$+ \ldots$$

$$S_{cd} = 2\,\mathrm{tr}\left[t^c S_n^+ t^a S_n \right] 2\,\mathrm{tr}\left[t^d S_{\bar{n}}^+ t^a S_{\bar{n}} \right]$$

$$\left(= 2\,\mathrm{tr}\left[t^c S_n^+ S_{\bar{n}} t^d S_{\bar{n}}^+ S_n \right] \right)$$

Taking the proton matrix element, all color-octet operators vanish, because the proton is a color singlet. Of the singlet operators, only

$$\langle p_1 | \bar{\chi}_1 \frac{\not{k}}{2} \chi_1 | p_1 \rangle$$

is nonvanishing.

The $\bar{\chi}_1 \frac{\not{k}}{2} \gamma_5 \chi_1$ matrix element vanishes because of parity conservation, the $\bar{\chi}_1 \frac{\not{n}}{2} \gamma_\perp^\mu \chi$ vanishes because it can only depend on the vectors $n^\mu, \bar{n}^\mu$ and P_1^μ and none of those has a $\perp$-component.

So, for the four-fermion case, we have shown that

$$W = \int ds \int dt \; C_{qq}(s, t, M^2) \; \bar{\chi}_1(s\bar{n}) \frac{\not{\bar{n}}}{2} \chi_1(0) \, \bar{\chi}_2(tn) \frac{\not{n}}{2} \chi_2(0)$$

+ "operators which have vanishing proton matrix elements"

+ "operators with $B_\perp$ fields"

Taking the quark matrix element, gives

$$\hat{W}_{qq} = \tilde{C}(\bar{n}\cdot p_1, n\cdot p_2, M^2) \; \bar{n}\cdot p_1 \; n\cdot p_2$$

and the proton matrix element gives the usual PDFs. After extending the discussion to the other operators, we get the result for W:

$$\frac{1}{4}\sum_{spin} \langle P_1 P_2 | W | P_1 P_2 \rangle$$

$$= \sum_{i,j} \int_{-1}^{1} \frac{d\xi_1}{\xi_1} \int_{-1}^{1} \frac{d\xi_2}{\xi_2} \; \hat{W}_{ij}(\xi_1, \xi_2, M^2)\, f_i(\xi_1) f_j(\xi_2)$$

where $i,j = g, u, d, s, \ldots$

As for DIS, the hadronic cross section is obtained by convoluting partonic cross sections with the PDFs.

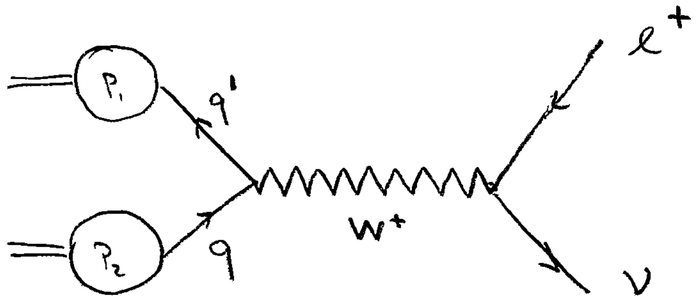
We have analyzed $d\sigma/dM^2$, but our analysis would work equally well for quantities which are more differential in the lepton momentum, such as

$$\frac{d\sigma}{dM^2\, dy\, dq_T}.$$

Here y is the photon rapidity $y = \frac{1}{2}\ln\left(\frac{q^0+q^z}{q^0-q^z}\right)$ and $q_T = \sqrt{q_x^2+q_y^2}$. Another common variable is the "transverse mass" $m_T^2 = M^2 + q_x^2 + q_y^2$

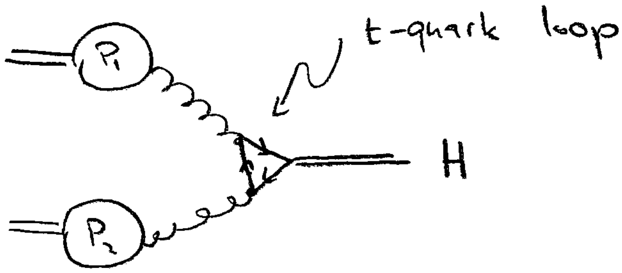
At higher energy, one needs to include both the photon and the Z-boson, to obtain the correct result for the cross section. (see figures)

A closely related process is W-production



the cross section is proportional to $|V_{qq'}|^2$.

Also Higgs production is rather similar



The Higgs boson coupling $\propto m_f$. The top contribution accounts for 95% of the signal,