7.3.1. Wilson lines and decoupling transformation

When matching the current operator in ϕ^3, we encountered non-local operators of the form $\phi_{c_1}(x+s\bar{n})\,\phi_{c_2}(x+tn)$. In a gauge theory a product of fields at different points is only gauge invariant if they are connected by a Wilson line. Define

the color matrices are ordered along the path.

$$[x+s\bar{n}, x] = \mathbb{P}\exp\left[ig\int_0^s ds'\,\bar{n}\cdot A(x+s'\bar{n})\right]$$

This object transforms as*

$$[x+s\bar{n}, x] \to V(x+s\bar{n})\,[x+s\bar{n}, x]\,V^\dagger(x)$$

So, an operator such as

$$\bar{\psi}(x+s\bar{n})\,[x+s\bar{n}, x]\,\psi(x)$$

is gauge invariant.

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* This is the transformation law of the link field $U(x,y)$ we used when we discussed gauge transformations.

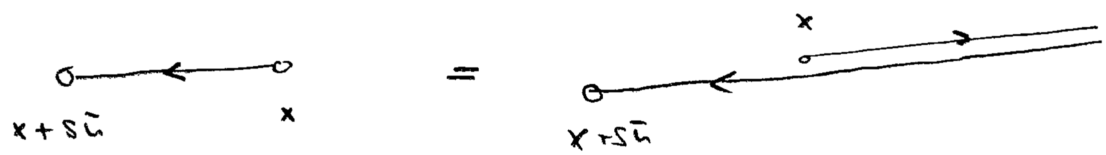
Instead of a line from x to $x + s\bar{n}$, it is customary to work with Wilson lines which run to infinity

$$W(x) = \mathbb{P} \exp\left[ig \int_{-\infty}^{0} ds \; \bar{n} \cdot A(x + s\bar{n}) \right]$$

$$= [x, -\infty]$$

A finite segment is

$$[x + s\bar{n}, x] = W(x + s\bar{n}) \, W^{+}(x)$$



If one restricts oneself to gauge transformations which vanish at infinity, the products

$$\chi(x) = W^{+}(x)\,\psi(x) \quad \text{and} \quad \bar{\chi}(x) = \bar{\psi}(x)\, W(x)$$

are gauge invariant and can be used as building blocks to construct non-local operators.

The Wilson line $W(x)$ fulfills

$$i\bar{n}\cdot D\, W(x) = 0 .$$

$$i\bar{n}\cdot D\, W(x) = (i\bar{n}\partial + g\bar{n}\cdot A)\, \mathbb{P}\exp\left[ig\int_{-\infty}^{0} ds\, \bar{n}A(x+s\bar{n})\right]$$

$$= (i\bar{n}\partial + g\bar{n} A)\, \mathbb{P}\exp\left[ig\int_{-\infty}^{n\cdot x/2} ds\, \bar{n}A\left(s\bar{n}^{\mu} + \bar{n}\cdot x\frac{n^{\mu}}{2} + x_{\perp}^{\mu}\right)\right]$$

$$= \left[i\frac{\bar{n}\cdot n}{2}\, ig\bar{n}A(x) + g\bar{n}A(x)\right] W(x) = 0$$

Two types of Wilson lines are important in SCET:

Collinear $\quad W_c(x) = \mathbb{P}\exp\left[ig\int_{-\infty}^{0} ds\, \bar{n}A_c(x+\bar{n}s)\right]$

Soft $\quad S_n(x) = \mathbb{P}\exp\left[ig\int_{-\infty}^{0} ds\, n\cdot A_s(x+ns)\right]$

The collinear lines are used to build operators, while the soft lines are useful because of the structure of the soft interaction.

The interaction between quarks and soft gluons has the form

$$\mathcal{L}_{c+s} = \bar{\xi}\,\frac{\not{\bar{n}}}{2}\, in\cdot D\, \xi$$

$$in\cdot D = in\cdot\partial + g n A_c(x) + g n\cdot A_s(x_-) \quad ; \; x_- = \bar{n}\cdot x \frac{n^\mu}{2}$$

Now perform a field redefinition:

$$\xi_c(x) \to S_n(x_-)\, \xi_c^{(0)}(x)$$

$$A_c^\mu(x) \to S_n(x_-)\, A_c^{(0)\mu}(x)\, S^\dagger(x_-)$$

$$\mathcal{L}_{c+s} = \bar{\xi}^{(0)} S_n^\dagger(x_-) \frac{\not{\bar{n}}}{2} n_\mu \left(i\partial^\mu + A_s^\mu(x_-) + S_n(x_-) A_c^\mu(x) S_n^\dagger(x_-)\right) S\xi$$

$$= \bar{\xi}^{(0)} S_n^\dagger(x_-) S_n^\dagger(x_-) \frac{\not{\bar{n}}}{2}\left[in\partial + g A_c^{(0)}(x)\right] \xi^{(0)}$$

$$= \bar{\xi}^{(0)}\, in\cdot D_c^{(0)} \frac{\not{\bar{n}}}{2}\, \xi^{(0)}$$

This is also called the <u>decoupling transformation</u>, since it decouples the soft gluons from the collinear Lagrangian.

This decoupling is an important ingredient to factorization, but does not imply that everything factorizes at leading power.

For example, the vector current in QCD matches at leading power onto

$$J^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$$

$$\to \int ds \int dt \; C(s,t)\, \bar{\chi}_1(x+s\bar{n})\gamma_\perp^\mu \chi_2(x+tn)$$

$$\chi_1 = W_{c_1}^+ \cdot \xi_{c_1} \qquad \not{n}\chi_1 = 0$$

$$\chi_2 = W_{c_2}^+ \xi_{c_2} \qquad \not{\bar{n}}\chi_2 = 0$$

$$\gamma^\mu = \not{n}\frac{\bar{n}^\mu}{2} + \not{\bar{n}}\frac{n^\mu}{2} + \gamma_\perp^\mu$$

The decoupling transformations are

$$\chi_1(x) \to S_n(x_-)\chi_1^{(0)}(x)$$

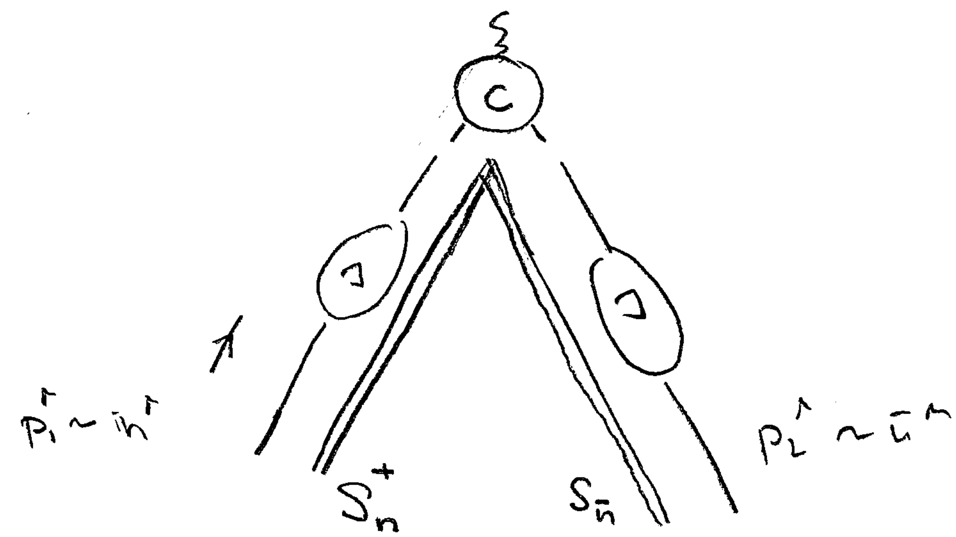
$$\chi_2(x) \to S_{\bar{n}}(x_+)\chi_2^{(0)}(x)$$

(for χ_2, we have to replace $n \leftrightarrow \bar{n}$)

So the current becomes

$$J^\mu(x) = \int ds \int dt\, C(s,t)\, \bar{\chi}_1^{(0)}(x+s\bar{n})\, S_n^\dagger(x_-) S_{\bar{n}}(x_+)\, \gamma_\perp^\mu \chi_2^{(0)}(x)$$

We managed to decouple the soft fields from the Lagrangian but they are still present in the operator J^μ. In contrast to ϕ^3 in $d=6$, the Sudakov form factor gets low energy contributions which describe a long-range interaction between the fast-moving in- and out-going quarks.



In this sence, it is non-factorizable: the low-energy contributions do not separate into corrections associated with the individual particles.