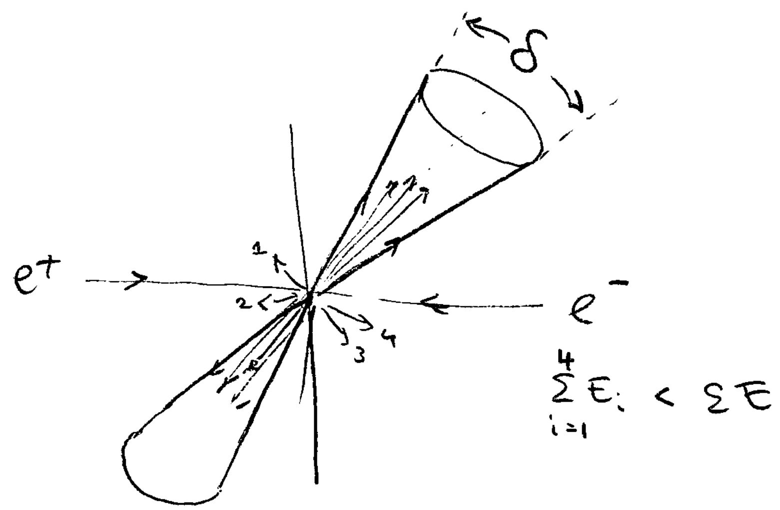
6. Event-shape variables and jets

We have seen that the total $e^+e^- \to$ hadrons cross section can be calculated in perturbation theory. We now want to consider more complicated observables. A minimal requirement for sensible observables is that the infrared divergences present in individual diagrams cancel. These arise from soft and collinear partons (quarks and gluons).

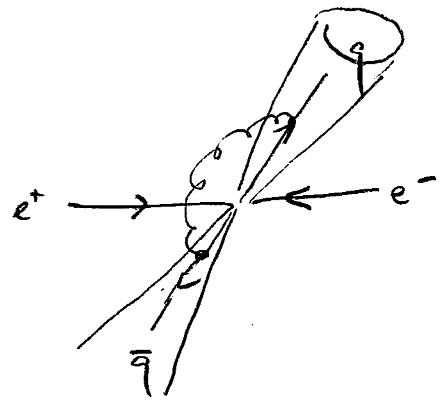
The first definition which fulfills this requirement was given by Sterman & Weinberg '77.
The cross section for two Sterman-Weinberg jets is defined as follows:



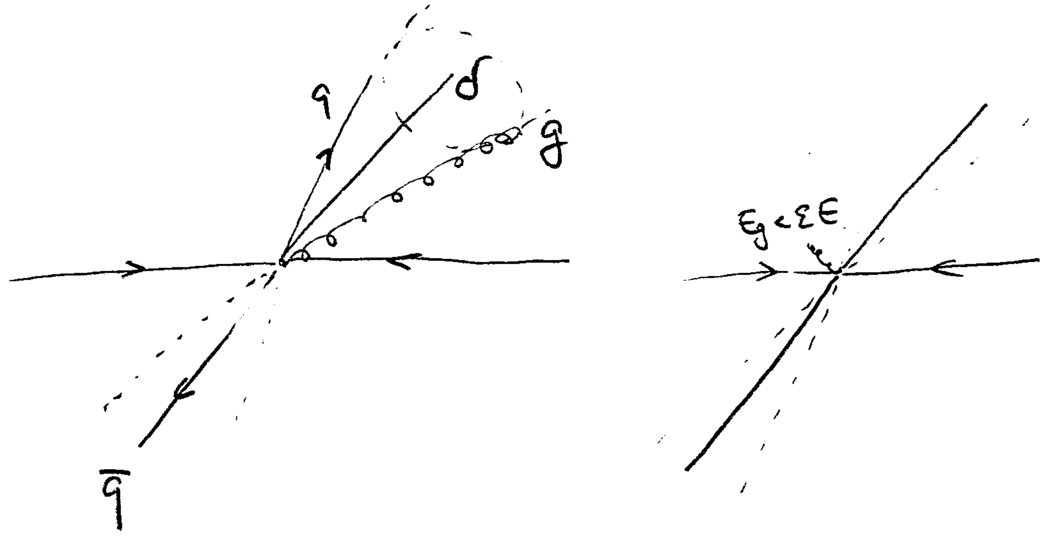
An event contributes if all the energy of its particles is contained in two cones of opening angles δ, except for a fraction ε of the total energy, so $\sigma_{SW} \equiv \sigma_{SW}(\delta, \varepsilon)$.

Let's see how this works for $e^+e^- \to \bar{q}q$ and $e^+e^- \to \bar{q}qg$

1.) All of the Born-level (lowest order) and all of the virtual corrections contribute, irrespective of δ and ε



2.) The real emission contributes, if the gluon energy is small $E_g < \varepsilon E$ or if the angle between the quark and gluon is smaller than δ.



So the Sterman Weinberg cross section includes <u>all</u> IR singular parts of the cross section.
Since the singularities cancelled for the total cross section, also σ_{SW} is finite.

The crucial feature of σ_{SW} is that it remains unchanged by very soft or very collinear emissions. The insensitivity to these low energy contributions should also guarantee that the results are not strongly affected by nonperturbative effects.

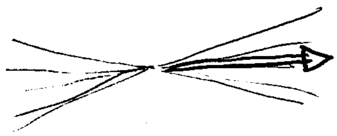
Many IR safe observables have been defined after the work of Sterman and Weinberg. Broadly speaking, they fall into two classes: event shapes and jet algorithms.

6.1. Event-shape variables

The classic shape variable is thrust defined as

$$T = \frac{1}{Q} \max_{\{\vec{n}\}} \sum_i |\vec{p}_i \cdot \hat{n}| \qquad Q = \sum_i |p_i|$$

The vector which maximizes T,
$\hat{n}_T$ is the thrust axis, the direction of maximum momentum transfer,



$\vec{n}_T$ (or the opposite direction)

Let's look at two extreme cases

1.) All particles have momenta in the same direction $\vec{p}_i = \pm |\vec{p}_i| \hat{n}_T$

"pencil-like"

$$T = \frac{1}{Q} \sum_i |\vec{p}_i| = 1$$

2.) The event is completely spherical. In this case any direction serves as the thrust axis. Let's chose $\hat{n}_T = (0, 0, 1)$.

We now have particles in any direction, all with the same momentum $|\vec{p}|$

So $\quad Q = \sum_i |p_i| = |\vec{p}| \int d\Omega = 4\pi |\vec{p}|$

$$T = \frac{1}{4\pi|\vec{p}|} \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\varphi \; |\vec{p}|\,|\cos\theta|$$

$$= \frac{1}{2} 2 \int_0^1 d\cos\theta \; \cos\theta = \underline{\underline{\frac{1}{2}}}$$

This is the minimum value of thrust.

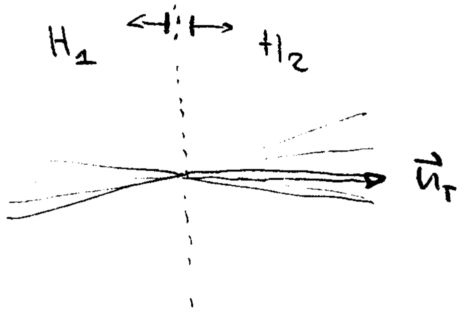
Exercise: What is the minimum value with 3 particles?

Thrust is infrared safe, since it is unchanged by soft and collinear emissions.

Let's consider a collinear splitting, $\vec{p} \to \vec{p}_A + \vec{p}_B$ with $\vec{p}_A \parallel \vec{p}_B$:

$$|\vec{p}\cdot\vec{n}_T| = |\vec{p}_A\cdot\vec{n}_T| + |\vec{p}_B\cdot\vec{n}_T| \quad \checkmark$$

With the thrust axis at hand, one can define several other shape variables. Define hemispheres:



* Hemisphere mass

$$M^2_{H_k} = \Big(\sum_{i\in H_k} |\vec{p}_i|\Big)^2 \qquad k = 1,2.$$

Heavy jet mass $\rho_H = \frac{1}{Q^2} \max\left(M^2_{H_1}, M^2_{H_2}\right)$

Light jet mass $\rho_L = \frac{1}{Q^2} \min\left(M^2_{H_1}, M^2_{H_2}\right)$

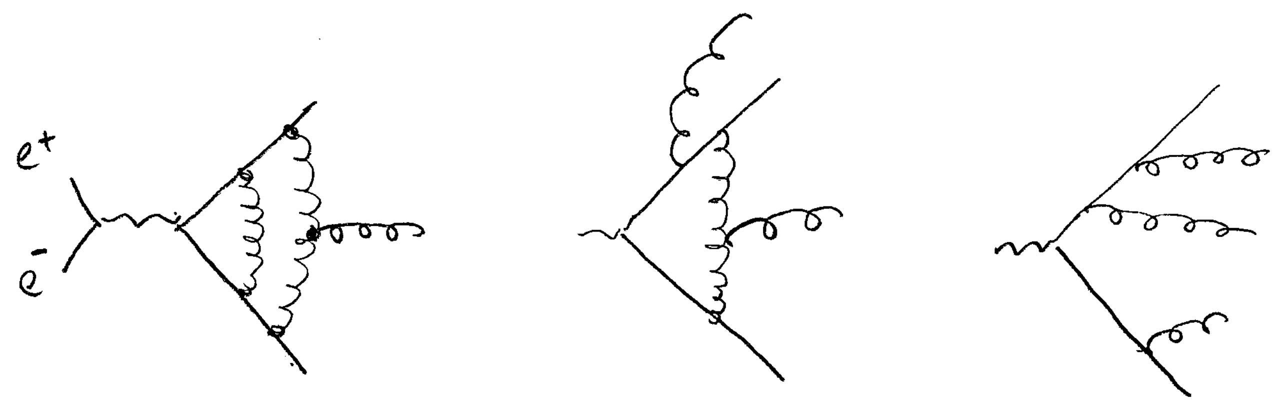
* Broadening: $B_{1,2} = \frac{1}{Q^2} \sum_{i\in H_{1,2}} \underbrace{|\vec{n}_T \times \vec{p}_i|}$

momentum transverse to the thrust axis.

Total broadening $B_T = B_1 + B_2$

Wide broadening $B_W = \max(B_1, B_2)$

These event shapes have recently* been calculated at NNLO. The relevant diagrams are



$\sigma_{q\bar{q}g}$ at 2 loops — $\sigma_{q\bar{q}gg}$ at 1 loop + many others — $\sigma_{q\bar{q}ggg}$ at tree level ⋮

* Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07
Weinzierl '08

Dealing with the IR div's which appear in the loop and phase-space integrals (and cancel in the sum of all contributions) is very nontrivial.

Comparing the calculated and measured distribution allows one to extract a value of α_s. (see figures).