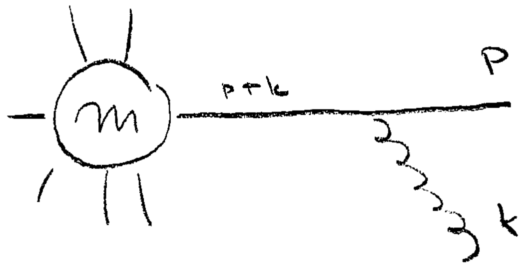
10.3. Soft emissions and coherent branching.

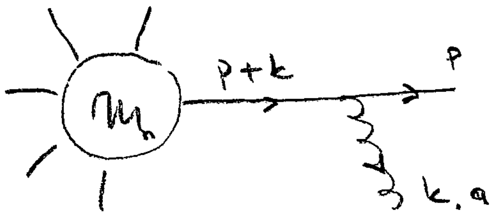
So far, we discussed the calculation of higher-order corrections from collinear emissions.



$$\frac{1}{(p+k)^2} = \frac{1}{2pk} = \frac{1}{2\omega E(1-\cos\Theta)}$$

These are enhanced, because the intermediate propagator goes on-shell for $\Theta \to 0$. However, the same enhancement also appears in the soft limit $\omega \to 0$. We have encountered this soft enhancement as $P(z) \sim \frac{1}{z}$ $z \to 0$ [or $P(z) \sim \frac{1}{1-z}$ for $z \to 1$] when we evaluated the splitting functions. Since both enhancements are logarithmic $\int_{z_0}^{z_1} dz \, \frac{1}{z} = \ln\left(\frac{z_1}{z_0}\right)$, $\int_{t_0}^{t_1} \frac{dt}{t} = \ln\left(\frac{t_1}{t_0}\right)$ it is important that a shower gets also the soft emissions correct, in particular those terms which are <u>both</u> collinear and soft enhanced.

The soft emissions factorize again on the amplitude level, e.g.



$$= \frac{ig\varepsilon^\mu}{2p\cdot k}\, i\bar{u}(p)\,\gamma_\mu(\not{p}+\not{k})\,\ldots$$

$$= -g\,\frac{\cancel{2}\,\varepsilon\cdot p}{\cancel{2}\,p\cdot k}\,(t^a)_{ij}\,(\mathcal{M}_n)_{\ldots j\ldots}$$

On the level of the cross section, this leads to

$$d\sigma_{n+1} = \frac{d\omega}{\omega}\,\frac{d\Omega}{2\pi}\,\frac{\alpha_s}{2\pi}\sum_{i,j}^{n} C_{i,j}\, W_{i,j}\, d\sigma_n$$

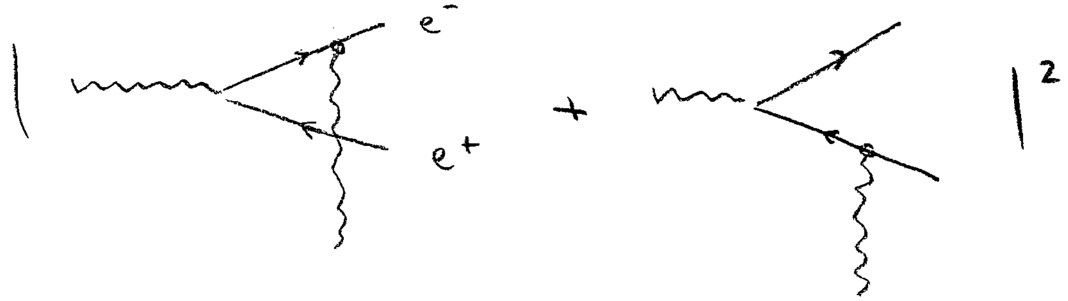
The sum is over external legs. C_{ij} is a color factor and

$$W_{i,j} = \frac{\omega^2\, p_i\cdot p_j}{p_i\cdot k\;\, p_j\cdot k} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})}$$

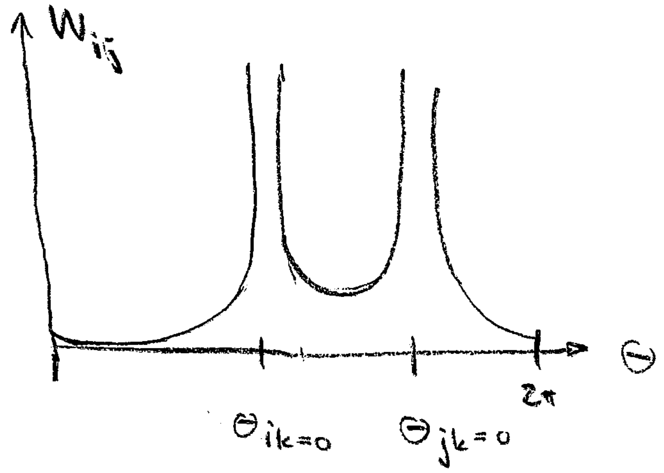
W_{ij} describes the interference of an emission from leg i with leg j.

This is a problem for the parton shower approach: we generate independent emissions from all legs, interference effects are missing.

In QED



At large angle the radiation interferes destructively. e^+e^- pair appears as an object of charge 0. Only at small angles individual charges are resolved.



Let us split W_{ij} into two parts

$$W_{ij} = W_{ij}^{[i]} + W_{ij}^{[j]}$$

where

$$W_{ij}^{[i]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{ik}} - \frac{1}{1-\cos\theta_{jk}} \right)$$

$$W_{ij}^{[j]} = \frac{1}{2}\left(\qquad - \qquad + \qquad \right)$$

The function $W_{ij}^{[i]}$ has a remarkable property.

If we write the angular integration as

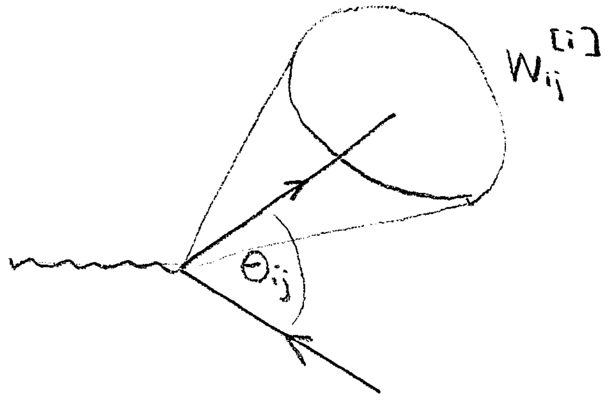
$$d\Omega = d\cos\theta_{ik}\, d\varphi_{ik}$$

then one can show that

$$\int_0^{2\pi} \frac{d\phi_{ik}}{2\pi} W_{ij}^{[i]} = \frac{1}{1-\cos\theta_{ik}} \qquad \text{if } \theta_{ik} < \theta_{ij}$$

$$= 0 \qquad \text{otherwise}$$

Averaged over the azimuthal angle, the soft radiation $W_{ij}^{[i]}$ has the same form as an emission from a single leg, except that there is no radiation outside an opening angle $\Theta_{iq} > \Theta_{ij}$.



The property that the azimuthally averaged radiation lies inside a cone is also called "angular ordering". For a proof that $W_{ij}^{[i]}$ has this property, see Ellis, Stirling & Webber.

This result implies that the parton shower gives the correct result if we use angular ordering: Instead of $t = p^2$, we should use the angle, or $1 - \cos\Theta$ as our ordering variable.

In this case, we obtain the correct pattern of soft emissions for quantities which are "azimuthally symmetric", i.e. which do not depend on <u>any</u> of the azimuthal angles of emissions.

There is one final complication: the ~~soft~~ emissions have color-structure $C_{ij} = -\sum_a T_i^a T_j^a = -T_i \cdot T_j$ where T_i and T_j are the color generators of particles i and j. Note that color-charge conservation implies $\sum_i T_i = 0$. Using this property $\sum_{j \neq i} T_i \cdot T_j = -T_i^2 = -C_i$ $(C_F \text{ or } C_A)$ (*)

The parton shower uses C_i for the emissions from leg i. The soft emissions have structure

$\sum_{\substack{i,j \\ i \neq j}}^{n} -T_i \cdot T_j W_{ij}$ so there are non-trivial color correlations. The replacement $T_i \cdot T_j \rightsquigarrow C_i$ is only correct in these cases

a.) for $n = 2$, since $T_1 = -T_2$; $T_1 \cdot T_2 = -T_1^2 = -T_2^2$

b.) for the leading soft + collinear log's, after azymuthal averaging.

c.) in the large-N_c limit

For b.) note that after the average $\int d\phi_{ik} W_{ik}^{[i]} = f(\cos\theta_{ki})$, independent of j, so we can use (*).

To summarize: an angle ordered shower produces the correct

1.) collinear logarithms

2.) soft + collinear double log's for quantities which are azymuthally symetric

3.) but does not produce soft logarithms correctly

After the shower stops at an IR cut-off scale, generators like Pythia, Herwig and Sherpa use hadronisation models to turn the partons into hadrons. We have discussed the various approximations which go into the construction of these codes. However, in general these programs do quite an amazing job in simulating high-energy collider events and are used extensively in particular by experimentalists.