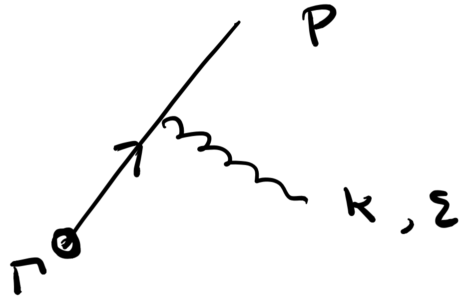
1.)



$$\to \quad e\, \bar{u}(p)\, \not{\varepsilon}^{*} \frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\,\Gamma$$

$$= e\, \bar{u}(p)\, \not{\varepsilon}^{*} \frac{\not{p}+m}{2p\cdot k+i\varepsilon}\,\Gamma \;+\; \cdots$$

$$= e\, \bar{u}(p) \left[\overbrace{(-\not{p}+m)}^{=0}\, \not{\varepsilon}^{*} + 2\varepsilon^{*}\cdot p \right] \frac{1}{2p\cdot k}\,\Gamma$$

$$= e\, \frac{\varepsilon^{*}\cdot p}{k\cdot p}\, \bar{u}(p)\,\Gamma$$

For diagrams with multiple emissions, one repeats the same steps.

2.)

Note that the field operator has the form

$$\hat{A}_\mu(x) = \sum_{\lambda=0}^{3} \int \frac{d^3\vec{k}}{(2\pi)^3\, 2E_k} \Big[\varepsilon_\mu(k,\lambda)\, \hat{a}(k,\lambda)\, e^{-ikx} + \varepsilon^*_\mu(k,\lambda)\, \hat{a}^+(k,\lambda)\, e^{ikx} \Big]$$

where a and a^+ are creation and annihilation operators for photons.

$$\hat{a}^+(k,\lambda)\,|0\rangle = |\vec{k},\lambda\rangle$$

$$S_1(q) = \langle \vec{q},\lambda|\, 1 - ie\int_0^\infty ds\, v\cdot A(v\cdot s)\, e^{-\delta s}\, |0\rangle$$

$$= -ie \sum_{\lambda'=0}^{3} \int_0^\infty ds \int \frac{d^3k}{2E_k(2\pi)^3}\, v\cdot\varepsilon^*(\lambda',k)\, e^{isv\cdot k}\, e^{-\delta s} \cdot \underbrace{\langle \vec{q},\lambda|\vec{k},\lambda'\rangle}_{(2\pi)^3\, 2E_k\, \delta^{(3)}(\vec{k}-\vec{q})\, \delta_{\lambda\lambda'}}$$

3.) See 1410.1892, section 2.1 for a detailed discussion!

a.) Hard region $k \sim M$

$$\frac{1}{k^2+m^2} = \frac{1}{k^2}\left\{1 - \frac{m^2}{k^2} + \cdots\right\}$$

$$\int_0^\infty dk \, \frac{k^{1+\varepsilon}}{k^2(k^2+M^2)} = \ldots$$

b.) Soft region $k \sim m$

$$\frac{1}{k^2+M^2} = \frac{1}{M^2}\left\{1 - \frac{k^2}{M^2} + \cdots\right\}$$

$$\int_0^\infty dk \, \frac{k^{1+\varepsilon}}{(k^2+m^2)M^2} = \ldots$$

$$= -ie\, v\cdot\varepsilon^*(\vec{q},\lambda) \int_0^\infty ds\, e^{isv\cdot k - s\delta}$$

$$= -ie\, v\cdot\varepsilon^* \cdot \frac{i}{v\cdot k + i\delta} = e\,\frac{\varepsilon^*\cdot v}{k\cdot v}$$

$$\equiv e\,\frac{p\cdot\varepsilon^*}{p\cdot k}\,. \quad ✓$$

similar for S_2.

4.)

$$P_+ = \frac{\not{n}\,\not{\bar{n}}}{4} = \frac{1}{4}\left(\overbrace{2n\cdot\bar{n}}^{4} - \not{\bar{n}}\,\not{n}\right)$$

$$= 1 - P_-$$

$$P_+^2 = \frac{\not{n}\,\not{\bar{n}}\,\not{n}\,\not{\bar{n}}}{16} = \frac{1}{16}\,\not{n}\left(\overbrace{2n\cdot\bar{n}}^{4} - \overset{=0}{\not{n}\,\not{n}}\right)\not{\bar{n}}$$

$$= P_+ \quad ✓$$

5.) $\Big[$

$$v^\mu = n\cdot v \frac{\bar{n}^\mu}{2} + \bar{n}\cdot v \frac{n^\mu}{2} + v_\perp^\mu$$

$$v_\perp^\mu := v^\mu - \bar{n}\cdot v \frac{n^\mu}{2} - n\cdot v \frac{\bar{n}^\mu}{2}$$

$$n\cdot v_\perp = n\cdot v - 0 - n\cdot v \underbrace{\frac{n\cdot\bar{n}}{2}}_{=1} = 0$$

$$\rightsquigarrow \quad \not{n}\gamma_\perp^\mu = -\gamma_\perp^\mu \not{n}$$

$\Big]$

$\Big[$ Note $\xi = P_+\psi = P_+^2\psi = P_+\xi$ $\Big]$

a.) $\not{n}\xi = \not{n}P_+\xi = 0$

b.) $\bar{\xi}\xi = \bar{\xi}\frac{\not{n}\not{\bar{n}}}{4}\xi = 0$

c.) $\bar{\xi}\not{D}_\perp\xi = \bar{\xi}\not{D}_\perp\frac{\not{n}\not{\bar{n}}}{4}\xi$

$$= \bar{\xi}\frac{\not{n}\not{\bar{n}}}{4}\not{D}_\perp\xi = 0$$

d.) $\bar{\xi}\gamma^\mu\xi = \bar{\xi}\gamma^\mu P_+\xi = \bar{\xi}\big(\overbrace{\frac{\not{n}}{2}\gamma^\mu}^{=0} - n^\mu\big)\frac{\not{\bar{n}}}{2}\xi$