1.)

a.)

$$SS^{+} = \mathbb{1}$$

$$\rightsquigarrow (\mathbb{1} + iT)(\mathbb{1} - iT^{+}) = \mathbb{1}$$

$$\rightsquigarrow i\underbrace{(T - T^{+})}_{i\,2\,\mathrm{Im}[T]} + TT^{+} = 0$$

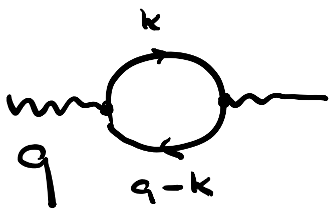
$$\rightsquigarrow 2\,\mathrm{Im}[T] = TT^{+} \quad \checkmark$$

b.)

$$2\,\mathrm{Im}\,\mathcal{M}(p_1 p_2 \to k_1 k_2)\,(2\pi)^4\,\delta^4(p_1 + p_2 - k_1 - k_2)$$

$$= \sum_x \mathcal{M}(p_1 p_2 \to k_x)\,\mathcal{M}^*(k_1 k_2 \to k_x)\,\underbrace{(2\pi)^4\,\delta(p_1 + p_2 - k_x)\,(2\pi)^4\,(k_1 + k_2 - k_x)}_{(2\pi)^4\,\delta^4(p_1 + p_2 - k_1 - k_2)\,\delta^4(p_1 + p_2 - k_x)}$$

2.)



$$= e^2 \sum_q e_q^2 N_c \cdot (-1) \cdot I$$

$$I = \mathrm{Im} \int \frac{d^d k}{(2\pi)^d} \frac{tr[\gamma^\mu \not{k} \gamma_\mu (\not{q} - \not{k})]}{(k^2 + i\varepsilon)((q-k)^2 + i\varepsilon)}$$

$$= \mathrm{Im} \int d^d k \, \frac{-8 k \cdot (q-k)}{\cdots}$$

$$= \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d r}{(2\pi)^d} (2\pi)^d \delta^d(q-k-r) \left(-8 k \cdot r\right)$$

$$\theta(k^0)\delta(k^2)\,\theta(r^0)\delta(r^2) \cdot (-i)^2 (2\pi)^2$$

$$= \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2E_k} \int \frac{d^3 r}{(2\pi)^3} \frac{1}{2E_r} \underbrace{8 \cdot k \cdot r}_{4q^2} \, \delta^{(4)}(q-k-r)\,(2\pi)^4$$

$$= \underbrace{\frac{1}{8\pi}}_{PS_2 \checkmark} \cdot 4q^2 = \frac{q^2}{2\pi}.$$

$$\left(q^2 = (k+r)^2 = 2k \cdot r \right)$$

$$\rightsquigarrow \mathrm{Im}\, \Pi_h{}^{\mu}{}_{\mu} = -e^2 \sum_q e_q^2 \cdot N_c \frac{s}{2\pi}$$

$$= 3\, s\, \mathrm{Im}\left[\Pi_h(s)\right]$$

$$\rightsquigarrow \mathrm{Im}\left[\Pi_h(s)\right] = -\frac{1}{6\pi} \overset{4\pi\alpha}{e^2} \sum_q e_q^2 N_c \quad \leftarrow \text{Factor } \frac{1}{2}\,?$$

$$\sigma = -\frac{4\pi\alpha}{s} \mathrm{Im}\left[\Pi_h\right] = \frac{8\pi\alpha^2}{3s} \sum_q e_q^2 N_c$$