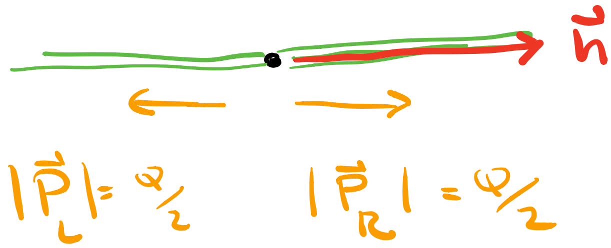
1.) use massless particles $Q = \sum_i |\vec{p}_i|$

b.)

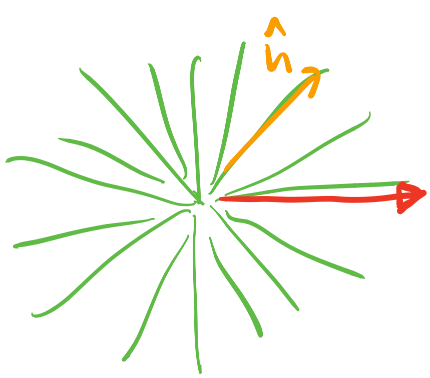


$$T = \frac{1}{Q}\sum |\vec{n}\cdot\vec{p}| = \frac{1}{Q}\Big(\underbrace{\vec{n}\cdot\vec{p}_R}_{} + \underbrace{|\vec{n}\cdot\vec{p}_L|}_{} \Big)$$

$$\underbrace{|\vec{p}_R|}_{Q/2} + \underbrace{|\vec{p}_L|}_{Q/2}$$

$$= 1$$

c.



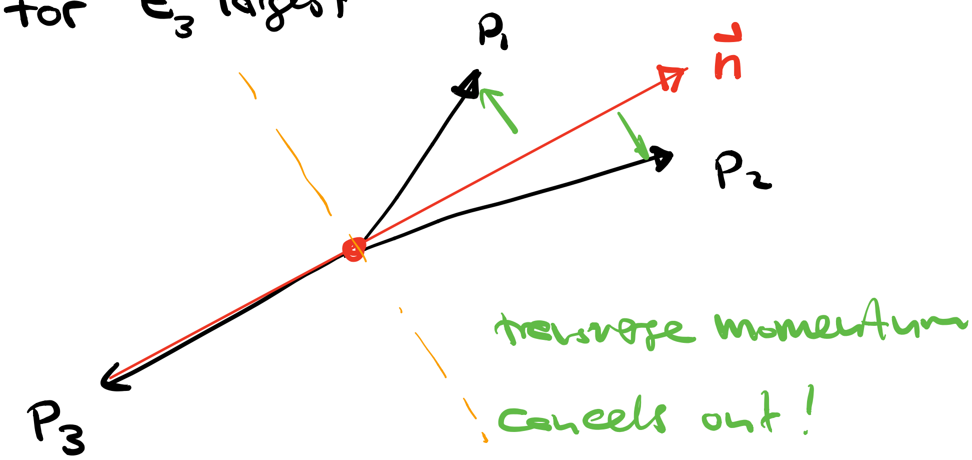
$\vec{p} = E(1, \hat{n})$

$\vec{n} = (0, 0, 1)$

$\hat{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$$\sum_i |\vec{p}\cdot\vec{n}| = \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\varphi \; E\cdot|\cos\theta|$$

2.) For E_3 largest



$$|\vec{n}\cdot\vec{p}_3| = E_3 \quad ; \quad \vec{n}\cdot(\vec{p}_1+\vec{p}_2) = |\vec{n}\cdot\vec{p}_3| = E_3$$

$$\tau = 1 - \frac{1}{Q}\sum_{i=1}^{3} |\vec{n}\cdot\vec{p}_i| .$$

$$\tau = 1 - \frac{1}{Q}\, 2E_3 \qquad \text{falls } E_3 > E_1, E_2$$

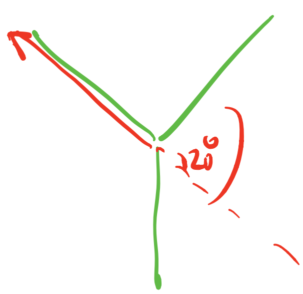
$$= y_3 = \frac{(p_1+p_2)^2}{Q^2} = \frac{s}{Q^2}$$

$$Q = \sum_i |p_i| = \int d\cos\theta \int d\varphi \cdot E = 4\pi \cdot E$$

$$\sum_i |\vec{p} \cdot \vec{n}| = 2\pi \cdot 2E \int_0^1 dz \, \overset{\cos\theta}{\overset{\parallel}{z}} = 4\pi E \cdot \frac{1}{2}$$

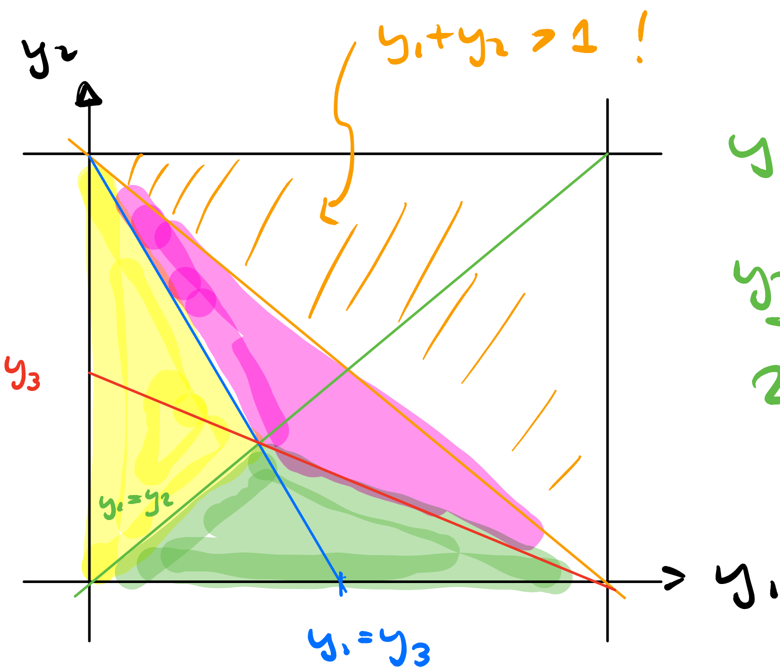
$$\leadsto T = \frac{1}{4\pi E} \cdot 4\pi E \, \frac{1}{2} = \frac{1}{2} .$$

3 particles



$$T = 1 - \frac{1}{3}$$

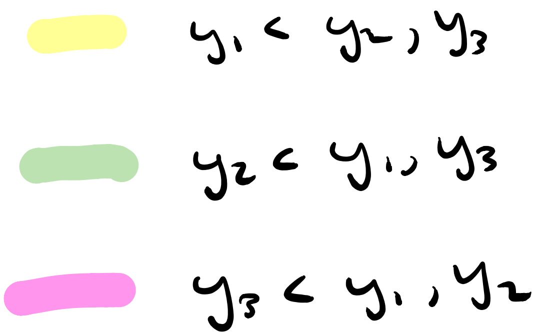
3.)



$y_3 + y_2 + y_1 = 1$

$\underline{y_3 = y_2:}$

$2y_1 + y_2 = 1$



Instead of parametrizing the different regions, it is easier to us Θ-functions to split the integral. Then compute each region in turn. For region $y_1 < y_2, y_1 < y_3$:

$$\underbrace{\int_0^1 dy_1 \int_0^{1-y_1} dy_2}_{\int_0^1 dy_2 \int_0^{1-y_2} dy_1} \; \Theta(y_2-y_1)\,\overbrace{\Theta(y_3-y_1)}^{\Theta(1-y_2-2y_1)}$$

$$(\ldots)\,\delta(\tau-y_1)$$

$$= \int_0^1 dy_2\, \Theta(1-y_2-\tau)\,\Theta(y_2-\tau)\,\Theta(1-y_2-2\tau)$$

$$(\ldots)\Big|_{y_1\to\tau}.$$

$$= \int_{\tau}^{1-2\tau} dy\, (\ldots\ldots)\Big|_{y_1\to\tau}$$

see Mathematica notebook for results.