1.)

$$\int \frac{d^{d-1}k_1}{2E_{k_1}(2\pi)^{d-1}} \int \frac{d^{d-1}k_2}{2E_{k_2}(2\pi)^{d-1}} (2\pi)^d \delta^{(d)}(q-k_1-k_2)$$

$$= \int \frac{d^{d-1}k_1}{2E_1(2\pi)^{d-1}} \frac{1}{2E_1} \frac{1}{(2\pi)} \delta(Q-2E_1)$$

$$= \int_0^\infty dk\, k^{d-2} \frac{1}{k^2} \frac{1}{(2\pi)^{d-2}} \frac{1}{4} \delta(Q-2k)$$

$$\cdot \int d^{d-1}\Omega$$

$$= \frac{\Omega_{d-1}}{8(2\pi)^{d-2}} \left(\frac{Q}{2}\right)^{d-4}.$$

$$= \frac{2^{-d}(\sqrt{\pi})^{3-d}}{\Gamma\left(\frac{d-1}{2}\right)} \left(\frac{Q}{2}\right)^{d-4}.$$

2.) 1.) Soft safety

2.) Collinear safety

a.) ~~1.~~ ~~2.~~

b.) 1. ✓ 2. ✓

c.) 1. ✓ ~~2.~~

(d.) 1. ✓ 2. ✓

3.) Definition:

$$\int_0^1 dx \left[f(x) \right]_+ \varphi(x)$$

$$:= \int_0^1 dx \, f(x) \left[\varphi(x) - \varphi(0) \right]$$

3.B $\int_0^1 dx \left(\frac{1}{x} \right)_+ \cos(x)$

$$= \int_0^1 dx \, \frac{1}{x} \left(\cos(x) - 1 \right)$$

$$x^{-1+\alpha} = \sum \frac{\alpha^n}{n!} \frac{\ln^n(x)}{x}$$

$$x^{\alpha} = \exp\left(\alpha \ln(x)\right)$$

4.)

$$I_1 = \int_0^1 dx \left(\frac{1}{\varepsilon} \delta(x) + \left[\frac{1}{x}\right]_+ + \varepsilon \left[\frac{\ln(x)}{x}\right]_+ \right)$$

$$(1-x)^2 \underbrace{e^{\varepsilon \ln(1-x)}}_{1 + \varepsilon \ln(1-x) + \varepsilon^2/2 \ln^2(1-x)}$$

$$= \int_0^1 dx \ \frac{1}{\varepsilon} \delta(x) + \left[\frac{1}{x}\right]_+ (1-x)^2 \left(1 + \varepsilon \ln(1-x)\right)$$

$$+ \varepsilon \left[\frac{\ln(x)}{x}\right] (1-x)^2$$

$$= \frac{1}{\varepsilon} + \int_0^1 dx\, \frac{1}{x} \Big[\underbrace{(1-x)^2 (1+\varepsilon \ln(1-x)) - 1}_{-2x + x^2 + \varepsilon (1-x)^2 \ln(1-x)} \Big]$$

$$+ \varepsilon \int_0^1 dx\, \frac{\ln x}{x} \Big[\underbrace{(1-x)^2 - 1}_{-2x + x^2} \Big]$$

$$\left[\text{Note:} \quad \int_0^1 dx\, \frac{1}{x} \ln(1-x) = -\frac{\pi^2}{6}. \right]$$

$$= \frac{1}{\varepsilon} - \frac{3}{2} + \left(3 - \frac{\pi^2}{6}\right) \varepsilon.$$

$$I_2 = \int_0^{\pi} d\theta\, |\sin\theta|^{-1+\varepsilon} \qquad \left(\begin{array}{l} z = \cos\theta \\ dz = \sin\theta\, d\theta \end{array} \right)$$

$$= \int_{-1}^{1} \overbrace{d\cos\theta}^{z}\ \overbrace{\frac{1}{|\sin\theta|} |\sin\theta|^{-1+\varepsilon}}^{|\sin\theta|^{-2-\varepsilon} \; = (1-z^2)^{-\frac{2+\varepsilon}{2}}}$$

$$= \int_{-1}^{1} dz \, \underbrace{(1-z^2)}_{(1-z)(1+z)}{}^{-1-\varepsilon/2}$$

$$= 2\cdot\int_{0}^{1} dz \, \overbrace{(1-z)}^{y=1-z}{}^{-1-\varepsilon/2} \, (1+z)^{-1-\varepsilon/2}$$

$$= 2\int_{0}^{1} dy \, y^{-1-\varepsilon/2} \, (2-y)^{-1-\varepsilon/2}$$

(Faktor :

$\cos\theta = 2\lambda - 1 \quad , \quad \lambda = 0 \ldots 1$)

$= \sim\sim\sim$

$$= -\frac{2}{\varepsilon} + 2\ln 2 + \varepsilon\left(\frac{\pi^2}{12} - \ln^2 2\right)$$

$$+ O(\varepsilon^2))$$