QCD — Exercise 8

Thrust — 26.11.20

1. At e^+e^- colliders the event shape variable thrust is defined as

$$T = \frac{1}{Q}\max_{\mathbf{n}}\sum_i |\mathbf{p}_i \cdot \mathbf{n}|\,, \tag{1}$$

with $\mathbf{n}^2 = 1$ and where the sum is over all momentum 3-vectors $\mathbf{p}_i$ in the event. The quantity Q is the center- of-mass energy of the collision.

(a) Show that thrust is IR safe.

(b) Show that $T = 1$ for the two-particle final state (and any "pencil-like" event involving only particles along a single direction).

(c) Show that $T = 1/2$ for a completely spherically symmetric event.

2. Consider $\tau = 1 - T$ for the three-particle final state $q(p_1)\,\bar{q}(p_2)\,g(p_3)$. Show that for three massless particles

$$\tau = \frac{1}{Q^2}\min(s,t,u)$$

where $s = (p_1 + p_2)^2$, $t = (p_1 + p_3)^2$, $u = (p_2 + p_3)^2$. *Hint:* The thrust axis points along the direction of the most energetic particle.

3. Compute the thrust distribution (for $\tau \neq 0$) using the 3-particle phase-space and amplitude squared derived in the lecture

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \frac{C_F\alpha_s}{4\pi}\int_0^1 dy_1 \int_0^{1-y_1} dy_2\, \frac{2\,(y_1^2 + y_2^2) + 4y_3}{y_1 y_2}\delta(\tau - \tau(y_1,y_2))\,,$$

where $\tau(y_1, y_2)$ is the value of thrust for given y_1 and y_2. According to the previous exercise this is

$$\tau(y_1,y_2) = \min(y_3, y_2, y_1)$$

with $y_3 = 1 - y_2 - y_2$.