QCD — Exercise 6

$e^+e^- \to q\,\bar{q}$ — 12.11.20

1. Compute the scattering amplitude $\mathcal{M}$ for the process $e^-(p_1)\,e^+(p_2) \to \bar{q}(p_3)\,q(p_4)$.

2. Square the amplitude $\mathcal{M}$, then average over the incoming spins and sum over the outgoing ones, i.e. compute the quantity

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 \,.$$

   Remember that

$$\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m\,, \qquad \sum_s v(p,s)\bar{v}(p,s) = \not{p} - m\,.$$

3. Show that the general formula for the cross section for the scattering process $p_1 + p_2 \to p_3 + \ldots p_n$,

$$d\sigma = \frac{1}{4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2}} \prod_{i=3}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}\, |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum_{k=3}^{n} p_k)\,,$$

   with $p_1^2 = p_2^2 = m^2$ reduces to

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\beta\,|\mathcal{M}|^2$$

   in the center-of-mass frame of the collision for $2 \to 2$ scattering with massless incoming particles. The quantity $\beta = |\vec{p}|/E$ is the velocity of the outgoing particles.

4. Using the results of the previous exercises, compute the total, spin-averaged cross section for $e^-(p_1)\,e^+(p_2) \to \bar{q}(p_3)\,q(p_4)$ neglecting the electron mass.