QCD Exercise 5

Chiral Perturbation Theory 29.10.20 and 5.11.20

1. If we consider QCD with *four* massless flavors, how many Goldstone bosons would arise in the breaking of chiral symmetry?

2. In the lecture, we encountered the relation (no summation over a)

$$\langle 0|\,[Q_A^a, P^a(y)]\,|0\rangle = -\frac{1}{n_f}\langle 0|\,\bar{\psi}(0)\psi(0)\,|0\rangle = -\langle 0|\,\bar{u}(0)u(0)\,|0\rangle\,,$$

   where n_f is the number of massless quarks, Q_A^a the axial charge, and

$$P^a(y) = \bar{\psi}(y)t^a\gamma_5\bar{\psi}(y)\,.$$

   Together with the Goldstone theorem, this implies that QCD has pseudoscalar Goldstone bosons, if the quark condensate is nonvanishing. Derive this relation using the canonical equal-time anti-commutation relations for the quark fields. The relation

$$[ab, cd] = a\{b,c\}d - ac\{b,d\} + \{a,c\}db - c\{a,d\}b$$

   is useful.

3. The effective Lagrangian for the Goldstone bosons associated with spontaneous chiral symmetry breaking takes the form

$$\mathcal{L} = \frac{F^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U\rangle + \mathcal{O}(p^4)\,, \tag{1}$$

   where $\langle \dots \rangle$ denotes the trace and the field $U(x)$ can be parameterized in terms of the Goldstone field π^a as

$$U(x) = \exp\left(\frac{2i}{F}\pi^a t^a\right)\,,$$

   The generators t^a correspond to the broken symmetries. Expand the Lagrangian to fourth order in the Goldstone fields π^a. For simplicity, work with two massless flavors, where $t^a = \sigma^a/2$.

4. Derive the Feynman rules for the Goldstone propagator and for the interaction of four Goldstone bosons.

5. Derive the Goldstone-boson scattering cross section, i.e. the cross section for the process

$$\pi^a(p_1)\,\pi^b(p_2) \to \pi^b(p_3)\,\pi^c(p_4)\,.$$

6. To construct the effective Lagrangian, we added external sources for left-handed and right-handed vector currents to the QCD Lagrangian and transformed these as

$$\begin{aligned} r_\mu &\to V_R\, r_\mu\, V_R^\dagger - i(\partial_\mu V_R)\, V_R^\dagger \\ l_\mu &\to V_L\, l_\mu\, V_L^\dagger - i(\partial_\mu V_L)\, V_L^\dagger \end{aligned}$$

   (a) Show that with these transformations of the external fields the QCD Lagrangian becomes invariant under local chiral transformations $V_L(x)$ and $V_R(x)$.

   (b) Show that the leading-power chiral perturbation theory Lagrangian (1) is invariant under local transformations if we replace the regular derivatives by covariant one

$$i\partial_\mu \to iD_\mu = i\partial_\mu U + r_\mu\, U - U\, l_\mu\,.$$