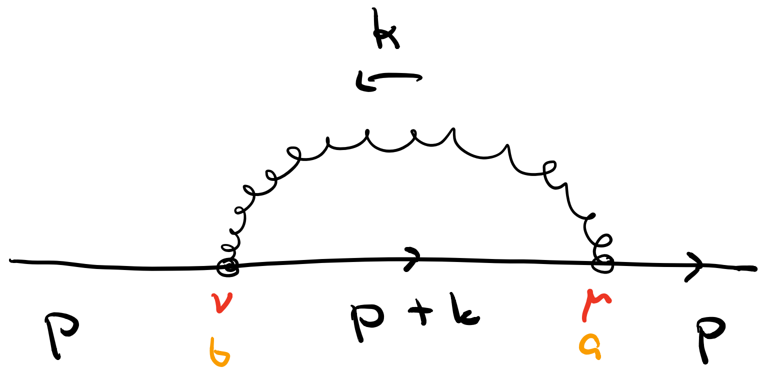
1.)



The amputated self-energy diagram is

$$-i\Sigma(p) = \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2+i\varepsilon}\left\{-g_{\mu\nu} + \xi\frac{k_\mu k_\nu}{k^2}\right\}\delta^{ab}$$

$$ig\gamma^\mu t^a \cdot \frac{i}{\not p + \not k - m + i\varepsilon}\, ig\gamma^\nu t^b$$

Color structure $t^a \cdot t^a = C_F \cdot \mathbb{1}$.

For simplicity, let's use $\xi = 0$ (Feynman gauge), we'll do the general case later using computer algebra.

$$-i\Sigma(p) = -g^2 C_F \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2[(k+p)^2-m^2]}$$

$$\cdot\, \gamma^\mu[\not p + \not k + m]\gamma_\mu$$

We need the Dirac structures

$$\gamma^\mu\gamma_\mu = \frac{1}{2}\{\gamma^\mu, \gamma_\mu\} = g^\mu{}_\mu = d$$

$$\gamma^\mu\gamma^\alpha\gamma_\mu = \gamma^\mu\overbrace{\{\gamma^\alpha, \gamma_\mu\}}^{2g^\alpha{}_\mu} - \overbrace{\gamma^\mu\gamma_\mu}^{d}\gamma^\alpha$$

$$= (2-d)\gamma^\mu$$

So

$$-i\Sigma(p) = -g^2 C_F \int \frac{d^dk}{(2\pi)^d} \frac{(2-d)(\not p + \not k) + d\cdot m}{k^2((k+p)^2-m^2)}$$

We thus need to compute two loop integrals

$$\{I, I_\mu\} = \int d^d k \, \frac{\{1, p^\mu + k^\mu\}}{k^2\left[(p+k)^2 - m^2\right]}$$

Now use Feynman parameters

$$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{\left[Ax + B(1-x)\right]^2} \qquad B = k^2, \quad A = (p+k)^2$$

$$\{I, I_\mu\} = \int_0^1 dx \int d^d k \, \frac{\{1, p^\mu + k^\mu\}}{\left[\underbrace{k^2 + 2x\, p\cdot k}_{(k+xp)^2 - x^2 p^2} + x p^2 - x m^2\right]^2}$$

Now shift $k \to k - xp$!

$$\{I, I_\mu\} = \int_0^1 dx \; \frac{\{1, \cancel{k^\mu}\xrightarrow{}0 \text{ (odd)} + (1-x)p^\mu\}}{[k^2 - M^2]^2}$$

with $M^2 = x m^2 - x(1-x)p^2$.

Now we can use the general dim. reg. formula

$$I(\alpha, \beta, M^2) = \int d^d k \; \frac{(k^2)^\alpha}{(M^2 - k^2 - i\varepsilon)^\beta} =$$

$$= i\pi^{d/2} (M^2)^{d/2+\alpha-\beta} \cdot \frac{\Gamma(\alpha+\frac{d}{2})\,\Gamma(\beta-\alpha-\frac{d}{2})}{\Gamma(d/2)\,\Gamma(\beta)}$$

We need $\alpha = 0$, $\beta = 2$; $d = 4 - 2\varepsilon$

$$I(0, 2, M^2) = i\pi^{d/2} (M^2)^{(d-4)/2} \; \Gamma(2 - d/2)$$

$$= i\pi^{d/2} (M^2)^{-\varepsilon} \Gamma(\varepsilon)$$

So we find

$$-i\Sigma(p) = -i \frac{\pi^{d/2}}{(2\pi)^d} \int_0^1 dx \left(x m^2 - x(1-x)p^2 + i\varepsilon \right)^{-\varepsilon}$$

$$g^2 C_F \Gamma(\varepsilon) \left\{ (4-2\varepsilon) m - (2-2\varepsilon) \not{p} (1-x) \right\}$$

For renormalization we need the divergent part of this expression.

To obtain it, we expand

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E + \cdots$$

$$\rightsquigarrow \Sigma(p) = + \frac{g^2 C_F}{(4\pi)^2} \frac{1}{\varepsilon} \int_0^1 dx \left\{ 4m - 2\not{p}(1-x) \right\}$$

$$= + \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} \left\{ -\not{p} + 4m \right\}$$

We will use this result to extract the mass & fermion field renormalizations.