Integrating out heavy flavors

Let us consider the effect of the top quark $m_t(\mu = m_t) \approx 163\,\text{GeV}$.

It is intuitively clear that there should be hardly any effects of this heavy particle at energies $Q \ll 2 m_t$.

Indeed, the results should be the same as QCD with $n_f = 5$ for $m_t \to \infty$, perhaps with slightly different $\alpha_s(\mu)$ and $m_f(\mu)$.

This intuition is correct. At lower energies, one can work with $QCD_{n_f=5}$. This is an example of the general method of Effective Field Theory (EFT). It should be possible to describe physics in terms of the low energy degrees of freedom.

One way to obtain the EFT would be to integrate out the top quark from the theory in the path integral. This would lead to a complicated determinant. It is much easier to simply write down the most general form of the

EFT and then determine the coefficients by computing the same physical quantity in the full and effective theory. To be sure that one reproduces the full result, one writes down the most general Lagrangian and orders the terms by dimension:

! No top quark!

$$\mathcal{L}_{EFT} = \sum_{i=1}^{5} \bar{\psi}(i\slashed{D} - m_f)\psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$

$$+ \sum_i C_i O_i$$

"Wilson coefficients" determined by matching

operators of dimension $d > 4$

Let us look at a concrete example:

$$O_{g1} = \mathrm{tr}[G^{\mu}{}_{\nu} G^{\nu}{}_{\rho} G^{\rho}{}_{\nu}]$$

d = 6

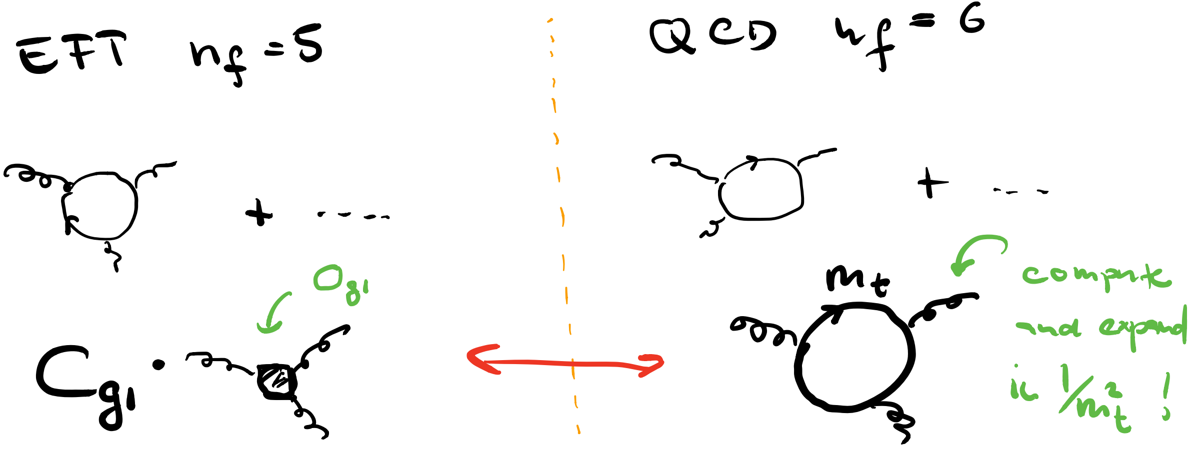
(note: there are several more d=6 operators!)

The Wilson coefficient C_{g1} therefore takes the form

dimensionless

$$C_{g1} = \frac{c_{g1}}{m_t^2}$$

and it's contribution is suppressed at low energies. To determine it, one computes



one then chooses C_{g1} so that the EFT result matches the full QCD result.

A similar matching computation is needed for the quark masses and the coupling constant.

see hep-ph/0908.4392 for a detailed discussion, or my EFT lecture notes at

ww.becher.itp.unibe.ch/eft/

The upshot of all the discussion is that one can simply work with $\mathcal{L}_{QCD}^{n_f=5}$ for energies below m_t, up to $\frac{1}{m_t^2}$ corrections.