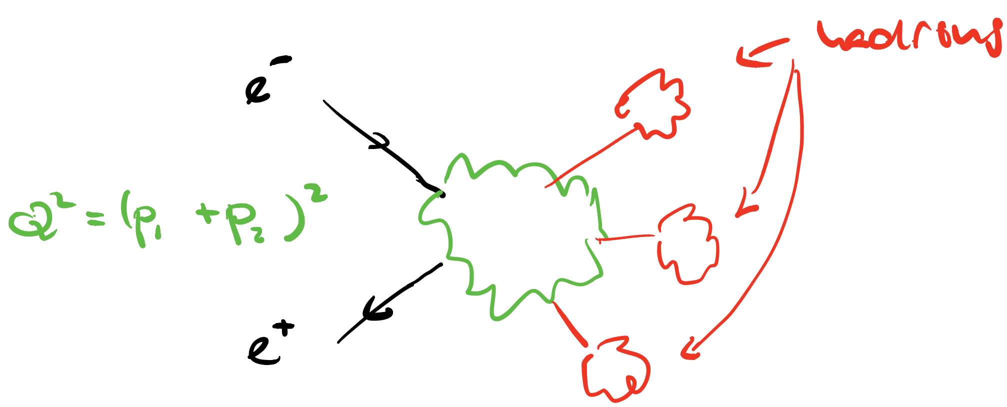
QCD at Colliders

Next, we turn to collider processes.
At first sight, it looks impossible to compute anything since the end-result are hadrons: non-perturbative, relativistic bound states of quarks & gluons:



Also, scattering processes are something deeply Minkowskian and it is unclear how to simulate them in lattice QCD.

Things are even worse at high collision energies Q^2. To numerically compute them would require a very fine lattice and a large volume.

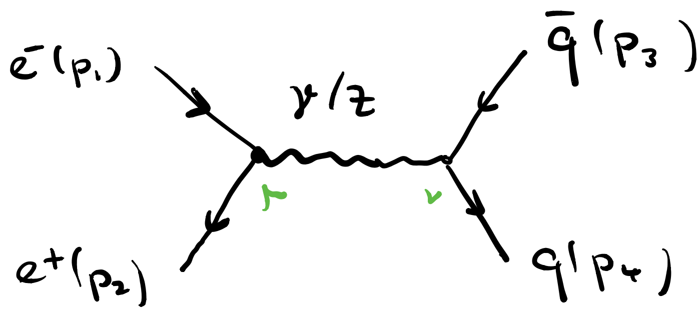
The key to analyze high energy collisions is factorization. One should separate the physics associated with Q^2 from the low energy physics at $\Lambda_{QCD}^2 \sim m_{Hadron}^2 \sim (GeV)^2$.

If this is successful, we can evaluate the high-energy part in perturbation theory.

The second important simplification is to compute observables that are insensitive to the low-energy bound-state dynamics, i.e. suitably inclusive cross sections.

$e^+e^- \to$ hadrons & the R-ratio

The simplest inclusive process we can consider is $e^+e^- \longrightarrow X$, where X is any hadronic final state. The high-energy part of this process is



+ "corrections"

Intuitively we expect that hadronisation effects should play a small role at large Q^2 since all the produced quarks and gluons will end up in hadrons, but our observable is completely insensitive to the type and the arrangement of them. We will later show that nonperturbative effects are suppressed by Λ_{QCD}^4 / Q^4 and up to these, one can compute the cross section in perturbation theory.

So let us compute the diagram shown above. The scattering amplitude for an intermediate γ is obtained as

on-shell wave function renormalizations

$Z_i = 1$ at lowest order

$$i\mathcal{M} = \left(\sqrt{Z_e}\right)^2 \left(\sqrt{Z_q}\right)^2$$

$$\times \; \bar{v}(p_2, m_e)(-ie\gamma^\mu) u(p_1, m_e)$$

$$\times \; \bar{u}(p_4, m_q)(+ie e_q \gamma^\nu) v(p_3, m_q) \; G_{\mu\nu}(q)$$

$e_u = +2/3$; $e_d = -1/3$, etc.

where $q^\mu = p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$ and

$$G_{\mu\nu}(q) = + \frac{i}{q^2}\left(-g_{\mu\nu} + \xi q^\mu q^\nu\right).$$

does not contribute

because $\not{p}_i u(p_i) = m_i u(p_i)$

$\not{p}_i v(p_i) = -m_i v(p_i)$

To get the cross section, we need

$$\frac{1}{2}\sum_{s_1} \cdot \frac{1}{2}\sum_{s_2}\sum_{s_3}\sum_{s_4} |\mathcal{M}|^2$$

to compute this, one uses

$$\sum_s u_s(p,m)\bar{u}_s(p,m) = \not{p} + m$$

$$\sum_s v_s(p,m)\bar{v}(p,m) = \not{p} - m$$

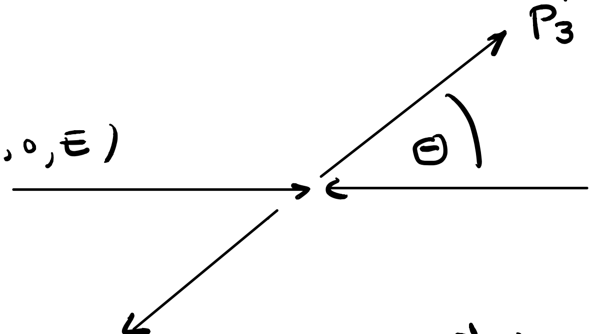
after which the squared amplitude reduces to a product of two traces. Neglecting the small electron mass, one obtains (exercise)

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8e^4 e_q^2}{q^4}\Big[p_1\cdot p_3\, p_2\cdot p_4 + p_1\cdot p_4\, p_2\cdot p_3 + m_q^2\, p_1\cdot p_2 \Big]$$

working in the center of mass frame, we have

$p_3^\mu = (E, \vec{k})$

$p_1^\mu = (E,0,0,E)$ $\qquad$ $p_2^\mu = (E,0,0,-E)$



$p_4^\mu = (E, -\vec{k})$

$$|\vec{k}| = \sqrt{E^2 - m_q^2} = E\beta$$

$$\rightsquigarrow 2p_1\cdot p_2 = (p_1+p_2)^2 = 4E^2 = q^2 = Q^2$$

$$p_1\cdot p_3 = E^2(1-\beta\cos\theta) = p_2\cdot p_4$$

$$p_1\cdot p_4 = E^2(1+\beta\cos\theta) = p_2\cdot p_3$$

$$\rightsquigarrow \frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = e^4 e_q^2\left(1+\frac{m_q^2}{E^2}+\beta^2\cos^2\theta\right)$$

And for 2 → 2 scattering with massless initial state particles one has

$$d\sigma = \frac{1}{2s}\int\frac{d^3p_3}{2E_3(2\pi)^3}\int\frac{d^3p_4}{2E_4(2\pi)^3}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4) \times|\mathcal{M}|^2$$

$$= \frac{1}{2s}\frac{1}{(4\pi)^2}\int d\Omega\int\frac{dk\,k^2}{E^2}\,\delta\left(2E-2\sqrt{k^2+m_q^2}\right)|\mathcal{M}|^2$$

β

$$= \frac{1}{4s}\frac{1}{(4\pi)^2}\,\frac{k}{E}\int d\Omega\,|\mathcal{M}|^2$$

Including the spin sums, we thus have

$$\frac{d\sigma}{d\Omega} = \frac{1}{4s} \frac{1}{(4\pi)^2} \beta \cdot \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2$$

$$= \frac{\alpha^2 e_q^2}{4s} \beta \left(1 + \frac{m_q^2}{E^2} + \beta^2 \cos^2\theta \right)$$

$$\int d\Omega \frac{d\sigma}{d\Omega} = 2\pi \int_{-1}^{1} d\cos\theta \frac{d\sigma}{d\Omega} = \frac{4\pi \alpha^2 e_q^2}{s} \beta \left[1 + \frac{m_q^2}{E^2} + \frac{\beta^2}{3} \right]$$

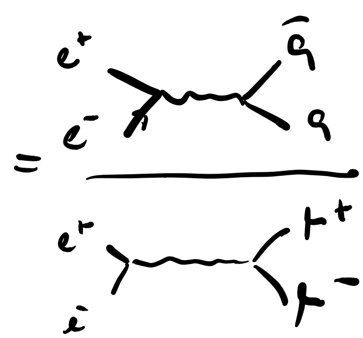
$$\rightsquigarrow \sigma_q = \frac{4\pi}{3} \frac{\alpha^2 e_q^2}{s} \sqrt{1 - \frac{m_q^2}{E^2}} \left(1 + \frac{m_q^2}{2E^2} \right)$$

We have computed the cross section for the production of an individual quark. To get the total cross section, we should sum over colors and all quarks that are light enough to be produced:

$$\sigma_{tot} = \sum_q N_c \cdot \sigma_q$$

To compare to experiment, it is nice to divide by $\sigma(e^+e^- \to \mu^+\mu^-)$, which is driven by the same leading-order diagram as $e^+e^- \to \bar{q}q$:

$$R = \frac{\sigma(e^+e^- \to \bar{q}q)}{\sigma(e^+e^- \to \mu^+\mu^-)} =$$



In fact, neglecting the masses, we can immediately write down the result for R:

$$R(s) = N_c \sum_q e_q^2 \cdot \left\{ 1 + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(m^2/s \right) \right\}$$

sum runs over quarks with $2m_q < \sqrt{s}$

For $\sqrt{s} < 2m_c^2$: $R = 3$

$2m_c^2 < \sqrt{s} < 2m_b^2$: $R = \frac{10}{3}$

$2m_b^2 < \sqrt{s} < 2m_t^2$: $R = \frac{11}{3}$

To compare to experiment at higher energies, we should also include the Z-boson contribution.

$\rightsquigarrow$ See slides for a comparison with experimental measurements