Chiral Perturbation Theory (CHPT)

We briefly encountered the concept of an Effective Field Theory (EFT) before, when discussing heavy flavors. The basic idea is that it should be possible to write down a theory with just the degrees of freedom that are present at a given energy. In QCD the low-energy degrees of freedom are hadrons (i.e. π's, K's, ..., $p, n, \ldots$), not quarks and gluons. At very low energy it should be possible to construct a Lagrangian involving only the lightest hadrons $\mathcal{L}_{eff} \equiv \mathcal{L}_{eff}(\pi^+, \pi^-, \pi^0)$.

By taking into account the

symmetries of QCD, one can organize $\mathcal{L}_{eff}$ and obtain a predictive framework: CHPT.

Chiral Symmetry

At low energies, we can integrate out the heavy quark flavors and use*

$$\mathcal{L}_{QCD}^{eff} = \sum_{f=u,d,s} \bar{\psi}_f (i\not{D} - m) \psi_f - \frac{1}{4} G^{\mu\nu,a} G_{\mu\nu,a} + O(1/m_\psi^2)$$

* For the symmetry arguments, we could also work with the full $\mathcal{L}_{QCD}$.

The theory simplifies further, if we take the chiral limit $m_f \to 0$. Since only the mass distinguishes the quarks, we can rotate one flavor into another, i.e. we have a symmetry

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow \psi' = \underset{\substack{\uparrow \\ \text{unitary } 3\times 3 \text{ matrix}}}{V} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

In fact, without the mass term, we have

$$\bar{\psi}\, i\not{D}\, \psi = \bar{\psi}_L\, i\not{D}\, \psi_L + \bar{\psi}_R\, i\not{D}\, \psi_R$$

with $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$; $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$.

We therefore can consider the independent unitary rotations

$$\psi_L \longrightarrow V_L \psi_L \; ; \; \psi_R \longrightarrow V_R \psi_R$$

Please note: V_L and V_R are global flavor rotations, while the local $SU(N_c)$ symmetry acts in the color space of the individual flavors.

Let us parameterize

$$V_{L,R} = \exp\left[i \underbrace{\alpha_{L,R}}_{U(1)} + i \underbrace{t^a \alpha^a_{L,R}}_{SU(n_f)} \right]$$

For two massless flavors $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ the generators are the Pauli matrices $t^a = \sigma^a/2$, $a = 1,2,3$.

With three massless flavors $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$, we have $t^a = \lambda^a/2$, $a = 1 \dots 8$, the Gell-Mann matrices.

For each symmetry, we obtain a classically conserved current, which according to Noether's theorem is

$$j^\mu \propto \frac{\delta \mathcal{L}}{\delta(\partial_\mu \psi)} \delta\psi = \bar{\psi} \gamma_\mu \cdot \delta\psi$$

We thus end up with currents:

$$L_\mu = \bar{\psi}_L \gamma_\mu \psi_L \quad ; \quad L_\mu^a = \bar{\psi}_L \gamma_\mu t^a \psi_L$$

$$R_\mu = \bar{\psi}_R \gamma_\mu \psi_R \quad ; \quad R_\mu^a = \bar{\psi}_R \gamma_\mu t^a \psi_R$$

It is also useful to consider the vector and axial currents

$$V^\mu = L^\mu + R^\mu = \bar{\psi} \gamma^\mu \psi$$

$$A^\mu = R^\mu - L^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

and analogously V_μ^a, A_μ^a.

It turns out that the axial current is anomalous, $\partial_\mu A^\mu \neq 0$ due to quantum effects. One can derive that

$$\partial_\mu A^\mu = \frac{\overset{\swarrow N_c = 3}{N_c} g_s^2}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} G^{\mu\nu,a} G^{\rho\sigma,a}$$

The remaining $SU_L(3) \times SU_R(3) \times U_V(1)$ transformations are a symmetry of the quantum theory and each current has an associated conserved charge

$$Q_k = \int d^3x \; j_k^0 \quad \text{mit} \quad [\underset{\text{QCD Hamilton-Operator}}{H}, Q_k] = 0 .$$

For 3 massless flavors, we have $2\times 8+1$ conserved charges Q_V, Q_V^a, Q_A^a.

When a theory has a symmetry, one should ask whether the spectrum is also symmetric. If not, one says that the symmetry is spontaneously broken. Vafa & Witten were able to show in '84 that vector-like symmetries are <u>not</u> spontaneously broken in vector-like theories like QCD, so only Q_A^a could be broken.

Let us consider the two possibilities in turn for $N_f = 3$ massless quarks.

A.) Unbroken symmetry $Q_A^a |0\rangle = 0$

spectrum contains degenerate multiplets of $G = SU_L(3) \times SU_R(3)$.

B.) Broken symmetry $Q_A^a |0\rangle \neq 0$

- Multiplets of $SU_V(3)$
- $N_f^2 - 1 = 8$ massless Goldstone bosons (one GB for each broken generator)

A careful derivation of Goldstone's theorem shows that if the correlator

$$\langle 0 | [\underbrace{Q(t)}_{\text{charge for broken symm}} , \underbrace{O(t, \vec{x})}_{\text{operator with quantum numbers of GB}}] | 0 \rangle$$

is nonzero, then the theory must contain a massless boson with quantum numbers of O

In QCD, one can choose $O = P^a = \bar{\psi}\gamma_5 t^a \psi$, the pseudoscalar current. One can show that (exercise)

$$\langle 0|[Q_A^a, P^a(x)]|0\rangle = \frac{-1}{N_f}\langle 0|\bar{\psi}(0)\psi(0)|0\rangle$$

$$= -\langle 0|\bar{u}u|0\rangle = -\langle 0|\bar{d}d|0\rangle = -\langle 0|\bar{s}s|0\rangle$$

$\uparrow$
$SU(3)_V$

The quantity $\langle 0|\bar{\psi}\psi|0\rangle = \langle 0|\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L|0\rangle$ is called the quark condensate and breaks chiral symmetry. A nonvanishing quark condensate implies that chiral symm. is broken and that there are 8 pseudoscalar GBs.

Of course, the quark masses are non-zero and chiral symmetry is not exact. On the other hand, since the u-, d-, s-masses are small, one can treat the symmetry breaking mass term as perturbation.

Looking at the QCD spectrum, one finds three mesons π^+, π^-, π^0 which are quite light $m_\pi \approx 140$ MeV. They have spin 0 and are pseudo-scalars. It is plausible that they are the $SU_V(2)$ triplet of "Goldstone bosons" associated with the breaking of chiral symmetry in the $\binom{u}{d}$ sector:

$$SU_L(2) \times SU_R(2) \longrightarrow SU_V(2)$$

Since the symmetry is explicitly broken by the small up- and down-quark masses, these pseudo Goldstone Bosons (pGB) acquire a small mass.