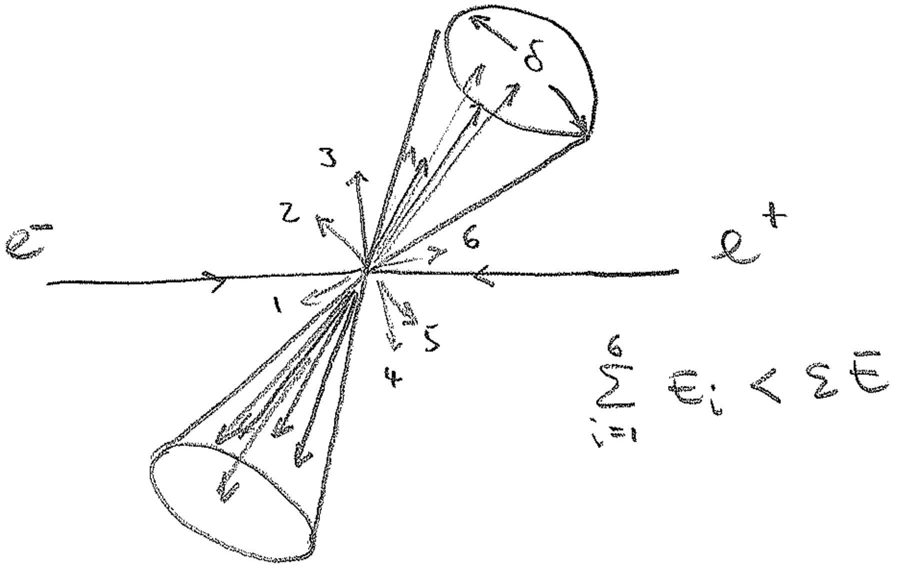
4. Jet physics

The result of particle collisions at high energy are typically jets: sprays of energetic hadrons into a region of the detector.

Instead of just computing total cross sections, we now want to predict jet rates. In the last chapter, we saw that the minimum requirement for a sensible observable is that it includes soft and collinear radiation. The first definition of jet cross sections is due to Sterman & Weinberg '77. They consider e^+e^- collisions and introduce two cones with opening angle δ, pointing in opposite directions.



An event contributes to the Sterman-Weinberg two-jet cross section $\sigma_{SW}(\delta, \varepsilon)$ if the fraction of energy outside the cones is smaller than ε.

Let us verify that this cross section is finite to $O(g_s^2)$, i.e. for $\sigma_{q\bar{q}} + \sigma_{q\bar{q}g}$

1.) $\sigma_{q\bar{q}}$ is part of $\sigma_{SW}(\delta, \varepsilon)$, irrespective of the values of $\varepsilon, \delta > 0$.

2.) Only part of the real-emission correction $\sigma_{q\bar{q}g}$ contributes. However the regions where two particles become collinear $\theta < \delta$, or one particle becomes soft $E_i < \varepsilon E$ are always included.

Because of this, the necessary cancellations of IR singularities between takes place:

$\sigma_{SW}(\delta, \varepsilon)$ is IR safe.

To check, whether a given observable is IR safe, one considers an event and then replaces a particle with energy E by

a.) two collinear particles, with $E_1 + E_2 = E$, or

b.) adds a soft particle with E_S to the event and considers $E_S \to 0$.

If neither operation changes the observable Γ for an arbitrary initial configuration), then the observable is IR safe.

The jet algorithms used today fall into two classes: 1.) Cone algorithms (e.g. SW)

2.) sequential recombination algorithms.

Cone algorithms group particles according to distance in position space, while the sequential algorithms are based on measures in momentum space.

4.1. Sequential algorithms

4.1.1. e^+e^- collisions

Simplest example for e^+e^- collisions is the JADE algorithm:

1.) For each pair of particles, compute

$$y_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{Q^2} \left[= \frac{(p_i-p_j)^2}{Q^2} \quad \text{for } p_i^2 = p_j^2 = 0 \right]$$

2.) Find minimum y_{min} of the y_{ij}'s.

3.) If $y_{min} < y_{cut}$, merge i & j into a new "particle" with $p = p_i + p_j$. Repeat from 1.

4.) Otherwise, declare all particles as jets and terminate.

The smaller y_{cut}, the more jets one will obtain.
The algorithm is infrared safe, since soft and collinear partons are clustered immediately.

A disadvantage of the Jade algorithm is that soft particles get clustered, even if they move in opposite directions. To avoid this, one can replace

$$d_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

"k_T - algorithm"
(for e^+e^-)

The use of $\min(E_i^2, E_j^2)$ ensures that soft particles are clustered with harder ones in similar directions.

[$E^2(1-\cos\theta) \approx E^2\theta^2/2 \approx p_T^2/2$ for small θ.]

4.1.2. Hadron colliders

The above algorithms must be modified to be used at hadron colliders because

1.) The collisions are not happening in the CMS: Q^2 is not known.

2.) Every collision has jets down the beam pipe (from the proton remnants).

Useful kinematic variables at hadron colliders are:

rapidity $y = \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right)$ (beam in z-dir.)

transverse momentum $p_t = p_x^2 + p_y^2$

azimuthal angle $\phi = \operatorname{arctg} \frac{p_y}{p_x}$

Rapidity differences are invariant under boosts along z-direction:

$$\begin{pmatrix} E' \\ p_z' \end{pmatrix} = \begin{pmatrix} \cosh(\alpha) & -\sinh(\alpha) \\ -\sinh(\alpha) & \cosh(\alpha) \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix}$$

$$\rightarrow y' = y - \alpha$$

Experimentalists also use the

pseudorapidity $\eta = -\ln(\tan(\Theta/2))$

$$= \frac{1}{2} \ln\left(\frac{|p| + p_z}{|p| - p_z}\right)$$

$[= y \text{ for } m = 0]$

The distance measure used at hadron colliders is

$$d_{ij} = \min\left(p_{T_i}^{2p}, p_{T_j}^{2p}\right) \frac{\Delta R_{ij}^2}{R^2}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

R : jet radius, free parameter.

$$d_{iB} = p_{T_i}^{2p} \quad \text{"distance to beam"}$$

$p = 1$: k_T-algorithm (Catani et al., Ellis and Soper '93)

$p = 0$: C/A algorithm (Wobisch, Wengler '99)

$p = -1$: anti-k_T (Cacciari, Salam & Soyez '08)

Clustering sequence:

1.) Compute d_{iB}'s, d_{ij}'s. Find minimum.

2.) If it is a d_{ij}, combine i and j, goto 1.

3.) If it is d_{iB}, remove i from list, declare it a jet.

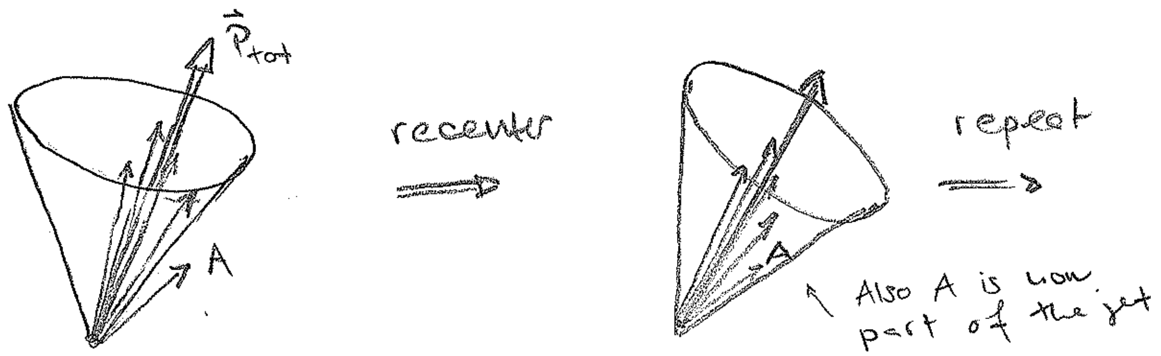
4.) Stop when no particles remain.

The anti-k_T algorithm is the default choice at the LHC, with $R \approx 0.5$. It's popularity is due to the fact that it produces very regular, cone-like jets, because it first clusters energetic collinear radiation and absorbs the soft radiation at the end. In contrast, the k_T algorithm produces quite irregular jet shapes, see Figures.

4.2. Cone algorithms

Despite the fact that cones have a more immediate physical interpretation, the cone algorithms are more complicated. In fact, all cone algorithms used at the Tevatron were IR unsafe and only in 2007 Salam and Soyez introduced an infrared safe algorithm, which is practical, the SISCone algorithm.

Modern cone algorithms are iterative. One first places a cone, then computes the total momentum of the particles inside the cone and recenters the cone around it.



The procedure is repeated until the cone is stable.

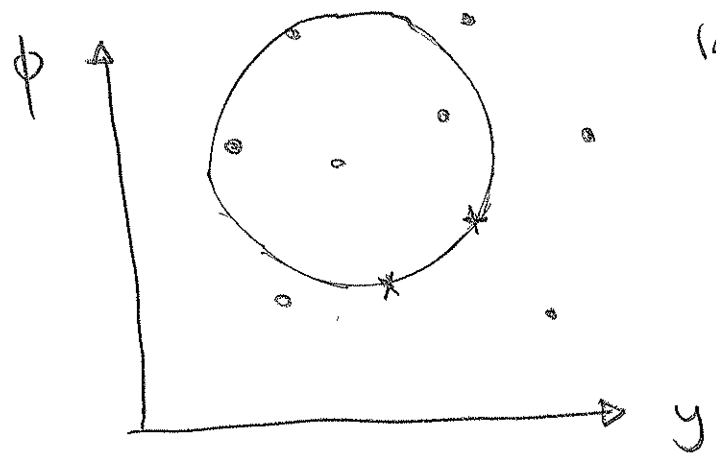
Two difficulties:

1.) What should be taken as the starting cones?

2.) What should be done if two stable cones overlap?

The solution to 1.) is to use a seedless algorithm, i.e. to consider all possible cones, i.e. use any subset of particles and check whether it leads to a stable cone. However, there are 2^N subsets so that this is not feasible for large N. $\Rightarrow$ Experimentalists used seeds, which spoils IR safety.

Salam and Soyez observation is that it is good enough to consider nearby points and that all stable cones can be obtained by considering all points within distance R of a given point p and then considering cones for which two points lie on the cone



$(\Delta y)^2 + (\Delta \phi)^2 < R^2$

See figures for further explanations. In the additional slides, also the procedure to split or merge overlapping cones is given.